

Formulas:

Time dilation; Length contraction: $\Delta t = \gamma \Delta t' \equiv \gamma \Delta t_p$; $L = L_p / \gamma$; $c = 3 \times 10^8 \text{ m/s}$

Lorentz transformation: $x' = \gamma(x - vt)$; $y' = y$; $z' = z$; $t' = \gamma(t - vx/c^2)$; inverse: $v \rightarrow -v$

Velocity transformation: $u_x' = \frac{u_x - v}{1 - u_x v/c^2}$; $u_y' = \frac{u_y}{\gamma(1 - u_x v/c^2)}$; inverse: $v \rightarrow -v$

Spacetime interval: $(\Delta s)^2 = (c\Delta t)^2 - [\Delta x^2 + \Delta y^2 + \Delta z^2]$

Relativistic Doppler shift: $f_{obs} = f_{source} \sqrt{1 + v/c} / \sqrt{1 - v/c}$

Momentum: $\vec{p} = \gamma m \vec{u}$; Energy: $E = \gamma mc^2$; Kinetic energy: $K = (\gamma - 1)mc^2$

Rest energy: $E_0 = mc^2$; $E = \sqrt{p^2 c^2 + m^2 c^4}$

Electron: $m_e = 0.511 \text{ MeV}/c^2$; Proton: $m_p = 938.26 \text{ MeV}/c^2$; Neutron: $m_n = 939.55 \text{ MeV}/c^2$

Atomic mass unit: $1 u = 931.5 \text{ MeV}/c^2$; electron volt: $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$

Stefan's law: $e_{tot} = \sigma T^4$, e_{tot} = power/unit area ; $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \text{K}^4$

$e_{tot} = cU/4$, U = energy density = $\int_0^\infty u(\lambda, T) d\lambda$; Wien's law: $\lambda_m T = \frac{hc}{4.96 k_B}$

Boltzmann distribution: $P(E) = C e^{-E/(k_B T)}$

Planck's law: $u_\lambda(\lambda, T) = N_\lambda(\lambda) \times \bar{E}(\lambda, T) = \frac{8\pi}{\lambda^4} \times \frac{hc/\lambda}{e^{hc/\lambda k_B T} - 1}$; $N(f) = \frac{8\pi f^2}{c^3}$

Photons: $E = hf = pc$; $f = c/\lambda$; $hc = 12,400 \text{ eV \AA}$; $k_B = (1/11,600) \text{ eV/K}$

Photoelectric effect: $eV_s = K_{max} = hf - \phi$, ϕ = work function; Bragg equation: $n\lambda = 2d \sin \theta$

Compton scattering: $\lambda' - \lambda = \frac{h}{m_e c} (1 - \cos \theta)$; $\frac{h}{m_e c} = 0.0243 \text{ \AA}$

Coulomb force: $F = \frac{kq_1 q_2}{r^2}$; Coulomb energy: $U = \frac{kq_1 q_2}{r}$; Coulomb potential: $V = \frac{kq}{r}$

Force in electric and magnetic fields (Lorentz force): $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$

Rutherford scattering: $\Delta n = C \frac{Z^2}{K_\alpha^2} \frac{1}{\sin^4(\phi/2)}$; $ke^2 = 14.4 \text{ eV \AA}$; $\hbar c = 1973 \text{ eV \AA}$

Hydrogen spectrum: $\frac{1}{\lambda_{mn}} = R \left(\frac{1}{m^2} - \frac{1}{n^2} \right)$; $R = 1.097 \times 10^7 \text{ m}^{-1} = \frac{1}{911.3 \text{ \AA}}$

Bohr atom: $E_n = -\frac{ke^2 Z}{2r_n} = -E_0 \frac{Z^2}{n^2}$; $E_0 = \frac{ke^2}{2a_0} = \frac{m_e (ke^2)}{2\hbar^2} = 13.6 \text{ eV}$; $K = \frac{m_e v^2}{2}$; $U = -\frac{ke^2 Z}{r}$

$hf = E_i - E_f$; $r_n = r_0 n^2$; $r_0 = \frac{a_0}{Z}$; $a_0 = \frac{\hbar^2}{m_e ke^2} = 0.529 \text{ \AA}$; $L = m_e v r = n\hbar$ angular momentum

de Broglie: $\lambda = \frac{h}{p}$; $f = \frac{E}{h}$; $\omega = 2\pi f$; $k = \frac{2\pi}{\lambda}$; $E = \hbar \omega$; $p = \hbar k$; $E = \frac{p^2}{2m}$

Wave packets: $y(x, t) = \sum_j a_j \cos(k_j x - \omega_j t)$, or $y(x, t) = \int dk a(k) e^{i(kx - \omega(k)t)}$; $\Delta k \Delta x \sim 1$; $\Delta \omega \Delta t \sim 1$

group and phase velocity : $v_g = \frac{d\omega}{dk}$; $v_p = \frac{\omega}{k}$; Heisenberg: $\Delta x \Delta p \sim \hbar$; $\Delta t \Delta E \sim \hbar$

Schrodinger equation: $-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + U(x)\Psi(x, t) = i\hbar \frac{\partial \Psi}{\partial t}$; $\Psi(x, t) = \psi(x) e^{-i\frac{E}{\hbar} t}$

Time-independent Schrodinger equation: $-\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + U(x)\psi(x) = E\psi(x)$; $\int_{-\infty}^{\infty} dx \psi^* \psi = 1$

∞ square well: $\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$; $E_n = \frac{\pi^2 \hbar^2 n^2}{2mL^2}$; $\frac{\hbar^2}{2m_e} = 3.81 \text{ eV} \cdot \text{\AA}^2$ (electron)

Harmonic oscillator: $\Psi_n(x) = H_n(x) e^{-\frac{m\omega}{2\hbar} x^2}$; $E_n = (n + \frac{1}{2})\hbar\omega$; $E = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}m\omega^2 A^2$; $\Delta n = \pm 1$

Expectation value of $[Q]$: $\langle Q \rangle = \int \psi^*(x)[Q]\psi(x) dx$; Momentum operator: $p = \frac{\hbar}{i} \frac{\partial}{\partial x}$

Eigenvalues and eigenfunctions: $[Q]\Psi = q\Psi$ (q is a constant); uncertainty: $\Delta Q = \sqrt{\langle Q^2 \rangle - \langle Q \rangle^2}$

Step potential: reflection coef: $R = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2}$, $T = 1 - R$; $k = \sqrt{\frac{2m}{\hbar^2}(E - U)}$

Tunneling: $\psi(x) \sim e^{-\alpha x}$; $T = e^{-2\alpha \Delta x}$; $T = e^{-2 \int_{x_1}^{x_2} \alpha(x) dx}$; $\alpha(x) = \sqrt{\frac{2m[U(x) - E]}{\hbar^2}}$

Schrodinger equation in 3D: $-\frac{\hbar^2}{2m} \nabla^2 \Psi + U(\vec{r})\Psi(\vec{r}, t) = i\hbar \frac{\partial \Psi}{\partial t}$; $\Psi(\vec{r}, t) = \psi(\vec{r}) e^{-i\frac{E}{\hbar} t}$

3D square well: $\Psi(x, y, z) = \Psi_1(x)\Psi_2(y)\Psi_3(z)$; $E = \frac{\pi^2 \hbar^2}{2m} \left(\frac{n_1^2}{L_1^2} + \frac{n_2^2}{L_2^2} + \frac{n_3^2}{L_3^2} \right)$

Spherically symmetric potential: $\Psi_{n, \ell, m_\ell}(r, \theta, \phi) = R_{n\ell}(r) Y_\ell^{m_\ell}(\theta, \phi)$; $Y_\ell^{m_\ell}(\theta, \phi) = P_\ell^{m_\ell}(\theta) e^{im_\ell \phi}$

Angular momentum: $\vec{L} = \vec{r} \times \vec{p}$; $[L_z] = \frac{\hbar}{i} \frac{\partial}{\partial \phi}$; $[L^2] Y_\ell^{m_\ell} = \ell(\ell + 1)\hbar^2 Y_\ell^{m_\ell}$; $[L_z] Y_\ell^{m_\ell} = m_\ell \hbar Y_\ell^{m_\ell}$

Radial probability density: $P(r) = r^2 |R_{n\ell}(r)|^2$; Energy: $E_n = -\frac{ke^2 Z^2}{2a_0 n^2}$

Ground state of hydrogen and hydrogen-like ions: $\Psi_{1,0,0} = \frac{1}{\pi^{1/2}} \left(\frac{Z}{a_0}\right)^{3/2} e^{-Zr/a_0}$

Orbital magnetic moment: $\vec{\mu} = \frac{-e}{2m_e} \vec{L}$; $\mu_z = -\mu_B m_\ell$; $\mu_B = \frac{e\hbar}{2m_e} = 5.79 \times 10^{-5} \text{ eV} / T$

Spin 1/2: $s = \frac{1}{2}$, $|S| = \sqrt{s(s+1)}\hbar$; $S_z = m_s \hbar$; $m_s = \pm 1/2$; $\vec{\mu}_s = \frac{-e}{2m_e} g\vec{S}$

Orbital + spin mag moment: $\vec{\mu} = \frac{-e}{2m_e} (\vec{L} + g\vec{S})$; Energy in mag. field: $U = -\vec{\mu} \cdot \vec{B}$

Two particles: $\Psi(\vec{r}_1, \vec{r}_2) = +/ - \Psi(\vec{r}_2, \vec{r}_1)$; symmetric/antisymmetric

Screening in multielectron atoms: $Z \rightarrow Z_{\text{eff}}$, $1 < Z_{\text{eff}} < Z$

Orbital ordering:

1s < 2s < 2p < 3s < 3p < 4s < 3d < 4p < 5s < 4d < 5p < 6s < 4f < 5d < 6p < 7s < 6d ~ 5f

Justify all your answers to all problems. Write clearly.