

Formulas:

Time dilation; Length contraction : $\Delta t = \gamma \Delta t' \equiv \gamma \Delta t_p$; $L = L_p / \gamma$; $c = 3 \times 10^8 \text{ m/s}$
 Lorentz transformation :

$$\begin{aligned} x' &= \gamma(x - vt) & x &= \gamma(x' + vt') \\ y' &= y, \quad z' = z & \gamma &= \frac{1}{\sqrt{1 - v^2/c^2}} & y &= y', \quad z = z' \\ t' &= \gamma(t - vx/c^2) & t &= \gamma(t' + vx'/c^2) \end{aligned}$$

Velocity transformation :

$$\begin{aligned} u_x' &= \frac{u_x - v}{1 - u_x v/c^2} & u_x &= \frac{u_x' + v}{1 + u_x' v/c^2} \\ u_y' &= \frac{u_y}{\gamma(1 - u_x v/c^2)} & u_y &= \frac{u_y'}{\gamma(1 + u_x' v/c^2)} \end{aligned}$$

Spacetime interval: $(\Delta s)^2 = (c\Delta t)^2 - [\Delta x^2 + \Delta y^2 + \Delta z^2]$

Relativistic Doppler shift: $f_{obs} = f_{source} \sqrt{1 + v/c} / \sqrt{1 - v/c}$

Momentum: $\vec{p} = \gamma m \vec{u}$; Energy: $E = \gamma mc^2$; Kinetic energy: $K = (\gamma - 1)mc^2$

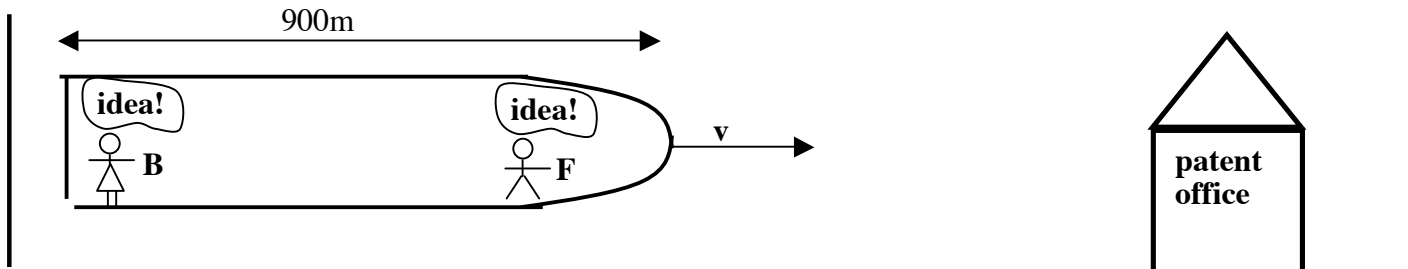
Rest energy: $E_0 = mc^2$; $E = \sqrt{p^2 c^2 + m^2 c^4}$

Electron: $m_e = 0.511 \text{ MeV}/c^2$ Proton: $m_p = 938.26 \text{ MeV}/c^2$ Neutron: $m_n = 939.55 \text{ MeV}/c^2$

Atomic mass unit : $1 u = 931.5 \text{ MeV}/c^2$; electron volt : $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$

Justify all your answers to all problems. Write clearly.

Problem 1 (10 points)



In a spaceship of proper length 900m traveling at speed v, Frank (F) sitting in the **front** suddenly has a great idea for a new product. Exactly $1 \mu\text{s}$ later (as measured by clocks on the spaceship), Bonnie (B) in the **back** of the ship comes up with the same idea. When the spaceship reaches the patent office on the ground, F and B run in to get a patent. Patent law says that the patent goes to whoever thought of the idea first. The patent officer decides according to clocks on the ground. Who gets the patent?

- If Frank gets the patent, how fast was the spaceship going? Give a range of v/c .
- If Bonnie gets the patent, how fast was the spaceship going? Give a range of v/c .
- Is it possible that B stole the idea from F? Justify clearly your answer.

Hint: use Lorentz transformation to do this problem.

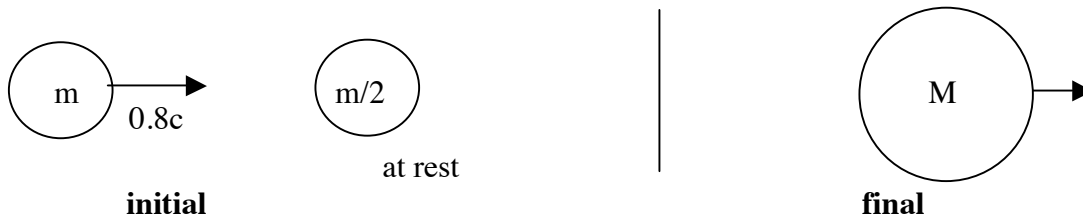
Problem 2 (10 points)



An electron and a proton are moving towards each other. The kinetic energy of the electron is 0.511 MeV and the kinetic energy of the proton is 938.26 MeV.

- (a) Find the speed of the electron, u_e . Give the answer as u_e/c .
- (a) Find the speed of the proton, u_p . Give the answer as u_p/c .
- (c) Find the speed of the proton relative to the electron. Give your answer in terms of c .

Problem 3 (10 points)



A particle of mass m moving at speed $0.8c$ hits another particle of mass $m/2$ at rest and they stick together. The resulting mass is M .

- (a) Find the speed at which M moves (in terms of c) (i) assuming classical (non-relativistic) mechanics is valid and (ii) using relativistic mechanics. Note: in classical mechanics mass is conserved.

Do (b), (c) and (d) using relativistic mechanics:

- (b) Find the value of M in terms of m .
- (c) Find the initial and final total kinetic energies and their difference, expressed as a factor times mc^2 .
- (d) Find the product of c^2 times the mass difference in the final and initial states, and compare with the difference in kinetic energies computed in (c).

Justify all your answers to all problems. Write clearly.