Physics 2D — Quiz 2 Solutions — Winter 2012

Problem 1

(a) $\lambda_m = \frac{hc}{4.96\, k_0 T} = \frac{12,400 \cdot 11,600}{4.96 \cdot 3000} \text{Å} \Rightarrow \boxed{\lambda_m = 9666.67\text{Å} = \lambda_0}$

(b) $T_0 \to T_1 = T_0/2$

We have: $\frac{hc}{\lambda_0 k_B T_0} = 4.96 \Rightarrow \frac{hc}{\lambda_0 k_B T_1} = 9.92$

power $\propto \frac{1}{e^{hc/\lambda k_B T} - 1}$, so $\frac{\text{power}(T_0)}{\text{power}(T_1)} = \frac{1}{e^{4.96}-1} \cdot (e^{9.92}-1) = 143.6$

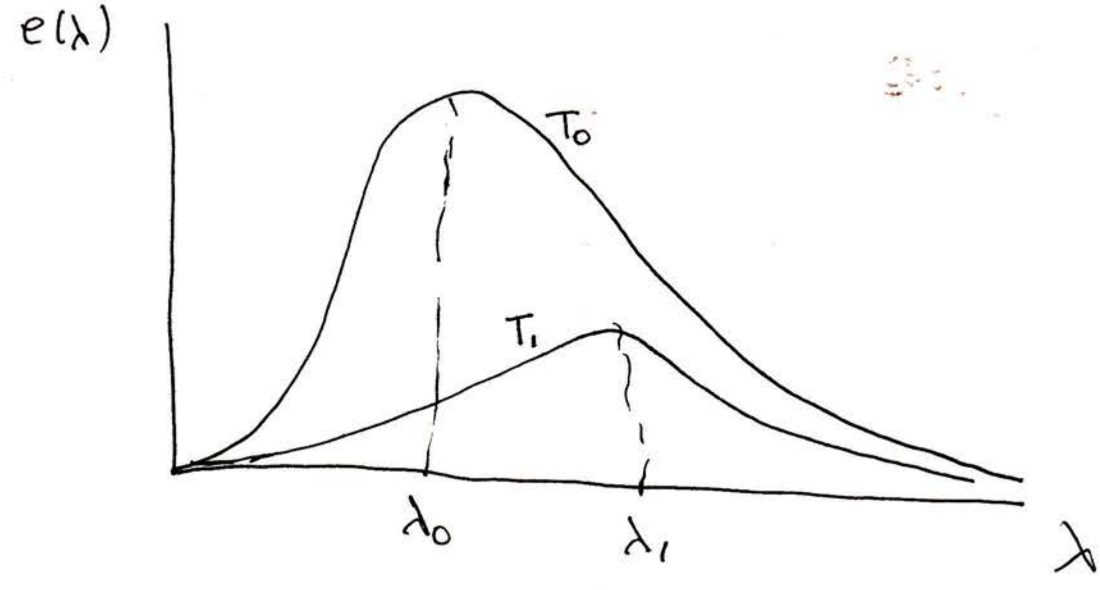
power decreases by a factor 143.6 when T changes from $T_0 = 3000\text{K}$ to $T_1 = 1500\text{K}$

(c) **Total power $\propto T^4 \Rightarrow$ total power decreases by factor $2^4 = 16$**

(d) For $\lambda \gg \lambda_m$, $\frac{1}{e^{hc/\lambda kT} - 1} \simeq \frac{1}{1 + \frac{hc}{\lambda k T} - 1} = \frac{\lambda k T}{hc}$

So for $\lambda_1 = 1000000\,\lambda_0$, power emitted decreases by factor of 2.

(e)



Problem 2

$K_{max} = 0.5\,eV$ for emitted electrons.

$hf = \frac{hc}{\lambda} = K_{max} - \phi$, for maximum K, λ should be minimum, so

$\lambda = 4000\,\text{Å} \Rightarrow \quad \phi = 0.5\,eV - \frac{12{,}400\,eV\text{Å}}{4000\,\text{Å}} = \boxed{2.6\,eV}$

(b) When light source moves away from metal, frequencies decrease

$\Rightarrow$ wavelengths increase according to

$$\lambda' = \lambda\sqrt{\frac{1+v/c}{1-v/c}}$$

The maximum wavelength that will cause photoemission is

$$\lambda_{max} = \frac{hc}{\phi} = \frac{12{,}400}{2.6}\,\text{Å} = 4769\,\text{Å}$$

The wavelengths in the range $4000\,\text{Å} \leq \lambda \leq 4500\,\text{Å}$ are Doppler-shifted up. When the maximum reaches λ_{max}, the number of ejected electrons will start to decrease, so

$$\lambda_{max} = \lambda_1\sqrt{\frac{1+v/c}{1-v/c}} \quad \text{with } \lambda_1 = 4500\,\text{Å} \Rightarrow$$

$$\frac{1+v/c}{1-v/c} = \left(\frac{\lambda_{max}}{\lambda_1}\right)^2 \Rightarrow 1 + \frac{v}{c} = \left(\frac{\lambda_{max}}{\lambda_1}\right)^2 - \left(\frac{\lambda_{max}}{\lambda_1}\right)^2\frac{v}{c} \Rightarrow$$

$$\frac{v}{c} = \frac{-1 + \left(\frac{\lambda_{max}}{\lambda_1}\right)^2}{1 + \left(\frac{\lambda_{max}}{\lambda_1}\right)^2} = \frac{-1 + 1.06^2}{1 + 1.06^2} = \frac{0.123}{2.123} = 0.058$$

$$\boxed{\frac{v}{c} = 0.058}$$

Problem 3

electron kinetic energy = difference in energy of incoming and outgoing photon

$$K_e = \frac{hc}{\lambda} - \frac{hc}{\lambda'} \quad , K_e = 50\,eV, \ \lambda' = 2\,Å$$

$$\frac{hc}{\lambda} = K_e + \frac{hc}{\lambda'} = 50\,eV + \frac{12{,}400\ eVÅ}{2\,Å} = 6250\,eV \Rightarrow \lambda = \frac{12{,}400}{6250}\,Å \Rightarrow$$

$\boxed{\lambda = 1.984\,Å}$. From $\lambda' - \lambda = \lambda_c(1-\cos\theta) \Rightarrow 1-\cos\theta = \frac{\lambda'-\lambda}{\lambda_c} \Rightarrow$

$$\cos\theta = 1 - \frac{\lambda'-\lambda}{\lambda_c} \quad ; \ \lambda_c = \frac{h}{m_e c} = \frac{12{,}400}{511{,}000}\,Å = 0.0243$$

$$\Rightarrow \cos\theta = 1 - \frac{2-1.984}{0.0243} = 0.342 \Rightarrow \boxed{\theta = 70°} \quad (\theta = 70.03°)$$

(b) The range of scattered wavelengths: $\lambda' - \lambda$ from 0 to $2\lambda_c$ for $0 \le \theta \le \pi$

$$\Rightarrow \text{for } \theta = \pi, \ \lambda' = \lambda + 2\lambda_c = 1.984\,Å + 2\lambda_c = 2.033\,Å$$

So $\boxed{1.984\,Å \le \lambda' \le 2.033\,Å}$

(c) For $\lambda' = \lambda$, $K_e = 0$; for $\lambda' = 2.033\,Å$, $K_e = 150.6\,eV$

So $\boxed{0 \le K_e \le 150.6\,eV}$