Physics 2D Quiz 3 Solutions Winter 2012

Problem 1

$\Delta n = C \cdot \frac{1}{\sin^4 \phi/2}$; for $\phi = 90°$, $\sin\frac{\phi}{2} = \sin 45° = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} \Rightarrow$

$\Delta n(90°) = C \cdot (\sqrt{2})^4 = 4C$

For $\phi = 180°$, $\sin\frac{\phi}{2} = \sin 90° = 1 \Rightarrow \Delta n(180°) = C \Rightarrow$

$\Delta n(90°)/\Delta n(180°) = 4$ assuming Rutherford's formula works.

For $K_\alpha = 19$ MeV, $\Delta n(90°)/\Delta n(180°) = 600/100 = 6 \Rightarrow$ formula doesn't work $\Rightarrow$ α-particle penetrates nucleus. For $K_\alpha = 18$ MeV, $\Delta n(90°)/\Delta n(180°) = 600/150 = 4 \Rightarrow$ formula works $\Rightarrow$ α-particle stays outside the nucleus even for $\phi = 180°$.

~~For 2ϕ~~ (a) For $K_\alpha = 17$ MeV < 18 MeV, Rutherford's formula will work $\Rightarrow$ there are $\boxed{\text{150 particles}}$ at $180°$ for every 600 particles at $90°$.

(b) For $K_\alpha = 18$ MeV, Rutherford's formula works also

$\Delta n(45°) = C \cdot \frac{1}{(\sin 22.5°)^4} = 46.6C \Rightarrow$ for every 600 particles

at $90°$ there are $600 \times \frac{46.6}{4} = \boxed{\text{6990 particles at } 45°}$

(c) Distance of closest approach d_{min}, calculate for $K_\alpha = 18$ MeV and 19 MeV

$\frac{k(Ze)(2e)}{d_{min}} = K_\alpha \Rightarrow d_{min} = \frac{2Zke^2}{K_\alpha} = \frac{2 \times 49 \times 14.4\,eV \cdot Å}{K_\alpha} \Rightarrow$

$d_{min}(K_\alpha = 18\,MeV) = 7.84 \times 10^{-5}$ Å ; $d_{min}(K_\alpha = 19\,MeV) = 7.43 \times 10^{-5}$ MeV

So we conclude for the radius of this nucleus R:

$$\boxed{7.43 \times 10^{-5}\,Å < R < 7.84 \times 10^{-5}\,Å}$$

Problem 2

$$hf = \frac{hc}{\lambda} = E_0 Z^2\left(\frac{1}{n^2} - \frac{1}{m^2}\right) \quad ; \; E_0 = 13.6\,eV, \; hc = 12400\,eV\,\mathring{A}$$

$$\Rightarrow \frac{1}{\lambda} = \frac{1}{911.3\mathring{A}} Z^2\left(\frac{1}{n^2} - \frac{1}{m^2}\right).$$ At room temperature, all ions are in the $n=1$ state. If $m=2$, $\frac{1}{n^2} - \frac{1}{m^2} = 1 - \frac{1}{4} = \frac{3}{4} \Rightarrow$

$$\Rightarrow \frac{1}{\lambda} = \frac{1}{911.3\mathring{A}} \cdot Z^2 \cdot \frac{3}{4} \Rightarrow \lambda = \frac{4}{3} \cdot \frac{911.3\mathring{A}}{Z^2} = \frac{1215.07\mathring{A}}{Z^2}.$$ For $Z=2$,

$$\lambda = \frac{1215.07\mathring{A}}{4} = 303.8\,\mathring{A} \Rightarrow \boxed{Z=2}$$

If $m>2$, λ is smaller for $Z=2$ no larger, and for $Z=1$, λ is larger than $911\,\mathring{A}$ for all transitions.

(b) For $Z=2$, and the transition $1\to3$:

$$\frac{1}{n^2} - \frac{1}{m^2} = 1 - \frac{1}{9} = \frac{8}{9} \Rightarrow \lambda = \frac{9}{8} \cdot \frac{911.3}{Z^2}\mathring{A} = 256.3\,\mathring{A}$$

For the transition $1\to4$: $\frac{1}{n^2} - \frac{1}{m^2} = 1 - \frac{1}{16} = \frac{15}{16} \Rightarrow \lambda = \frac{16}{15}\frac{911.3}{Z^2}\mathring{A} = 243.0\,\mathring{A}$

For $1\to5$, $\frac{1}{n^2} - \frac{1}{m^2} = 1 - \frac{1}{25} = \frac{24}{25} \Rightarrow \lambda = \frac{25}{24}\frac{911.3}{Z^2}\mathring{A} = 237.3\,\mathring{A}$

So for λ in the range $240\mathring{A} < \lambda < 500\mathring{A}$, $Z=2$, and all transitions starting at $n=1$ since atoms are in ground state, absorption lines are $\boxed{\lambda = 303.8\mathring{A}, 256.3\mathring{A}, 243.0\mathring{A}}$

(c) Emission lines: the three absorption wavelengths given above, plus the transitions $4\to3$, $4\to2$ and $3\to2$ $\Rightarrow$ $\boxed{\text{6 emission lines.}}$

$4\to3$: $\frac{1}{3^2} - \frac{1}{4^2} = \frac{7}{144} \Rightarrow \lambda = \frac{144}{7} \cdot \frac{911.3}{4}\mathring{A} = \frac{144}{7} \cdot 227.8\mathring{A} = \boxed{4687\mathring{A}}$

$4\to2$: $\frac{1}{2^2} - \frac{1}{4^2} = \frac{12}{64} \Rightarrow \lambda = \frac{16}{3} \cdot 227.8\mathring{A} = \boxed{1215\mathring{A}}$

$3\to2$: $-\frac{1}{3^2} + \frac{1}{2^2} = \frac{5}{36} \Rightarrow \lambda = \frac{36}{5} \cdot 227.8\mathring{A} = \boxed{1640\mathring{A}}$

Problem 3

$L = 4\hbar \Rightarrow$ electron is in the state $\boxed{n=4}$

If K, U are kinetic and potential energies, $K = -\frac{1}{2}U$, $U = -2K$,

$E = K + U = -K = -21.25\,eV$

$$E = -\frac{E_0 Z^2}{n^2} = \frac{-13.6\,eV \cdot Z^2}{4^2} = -21.25\,eV \Rightarrow$$

$$\Rightarrow Z^2 = +\frac{21.25 \times 4 \times 4}{13.6} = 25 \Rightarrow \boxed{Z=5}$$

(b) $L = m_e v r = n\hbar \Rightarrow v_n = \frac{n\hbar}{m_e r_n}$, $r_n = \frac{a_0}{Z} n^2 \Rightarrow$

$$v_n = \frac{n\hbar}{m_e \cdot a_0 \cdot n^2} \cdot Z = \frac{\hbar Z}{m_e a_0 n} \Rightarrow \frac{v_n}{c} = \frac{\hbar c}{m_e c^2 a_0} \frac{Z}{n} \Rightarrow$$

$$\Rightarrow \frac{v_n}{c} = \frac{1973}{0.511 \times 10^6 \cdot a_0} \cdot \frac{5}{4} \Rightarrow \boxed{\frac{v_n}{c} = 0.0091}$$ using $a_0 = 0.529\,\text{Å}$.

or $$\frac{v_n}{c} = \frac{\cancel{\hbar c}\, \cancel{m_e} k e^2}{\cancel{m_e} c^{\cancel{2}} \cdot \hbar^{\cancel{2}}} \frac{Z}{n} = \frac{k e^2}{\hbar c} \frac{Z}{n} = \frac{1}{137} \cdot \frac{Z}{n} = 0.0091$$

(c) $$r_n = a_0 \cdot \frac{4^2}{5} = 3.2\,a_0 = \boxed{1.69\,\text{Å}}$$

(d) $\mu = \frac{m_e \cdot M_{nucl}}{m_e + M_{nucl}}$; if $M_{nucl} = 5 m_e \Rightarrow \mu = \frac{5}{6} m_e$

The energies are proportional to the mass, so the energy would change by a factor 5/6 (we can assume $\mu = m_e$ for the hydrogen-like ion)

$E = K + U = -K = -21.25\,eV$

so energy instead of $-21.25\,eV$ would be

$$\boxed{-21.25\,eV \cdot \frac{5}{6} = -17.71\,eV}$$