Physics 2D — Quiz 4 Solutions — Winter 2012

Problem 1

$p = \frac{h}{\lambda}$, $p = m_e v \Rightarrow$ $v = \frac{h}{m_e \lambda} = \frac{hc}{m_e c^2 \lambda} c =$

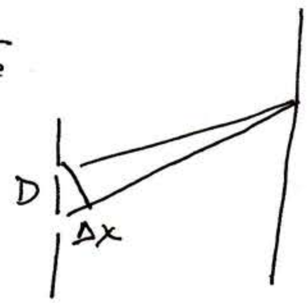
$$= \frac{12{,}400\,eV\,Å}{0.511 \cdot 10^6\,eV \cdot 2000\,Å} \cdot 3 \times 10^8\,\frac{m}{s} \Rightarrow \boxed{v = 3640\,m/s}$$

(b) The group velocity is the same as the particle velocity, $\boxed{v_g = 3640\,m/s}$

The phase velocity is half the group velocity, $\boxed{v_p = 1820\,m/s}$

because $\omega = \frac{\hbar k^2}{2m_e}$, $v_p = \frac{\omega}{k} = \frac{\hbar k}{2m_e}$, $v_g = \frac{d\omega}{dk} = \frac{\hbar k}{m_e}$

(c) The first minimum occurs when $\Delta x = \frac{\lambda}{2}$



$\Delta x = D \sin\theta = \frac{\lambda}{2} \Rightarrow$

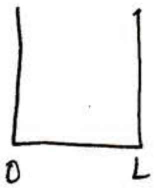
$$\sin\theta = \frac{\lambda}{2D} = \frac{2000\,Å}{2 \cdot 8000\,Å} = 0.125 \Rightarrow \boxed{\theta = 7.18°}$$

(d) $\psi_1 = Ae^{i\phi_1}$, $\psi_2 = Ae^{i\phi_2}$

for destructive interference, $\phi_2 = \phi_1 + \pi$, so that $e^{i\phi_2} = -e^{i\phi_1}$

So if $\phi_1 = \frac{\pi}{4} \Rightarrow \boxed{\phi_2 = \frac{\pi}{4} + \pi = \frac{5}{4}\pi}$

Problem 2



$L = 4\,\text{Å}$

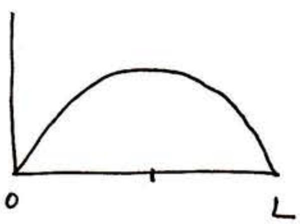
(a) $E_1 = \dfrac{\hbar^2\pi^2}{2m_e L^2} = \dfrac{1973^2\pi^2}{0.511\cdot10^6\cdot 4^2}\,eV \Rightarrow \boxed{E_1 = 2.35\,eV}$

(b) longest wavelength photon absorbed is $n=1 \rightarrow n=2$.

$E_2 = 2^2 E_1 = 4E_1 \Rightarrow E_2 - E_1 = 3E_1 = \dfrac{hc}{\lambda} \Rightarrow$

$\Rightarrow \lambda = \dfrac{hc}{3E_1} = \dfrac{12{,}400\,eV\,\text{Å}}{3\times 2.35\,eV} \Rightarrow \boxed{\lambda = 1758.9\,\text{Å}}$

(c) Wavefunction looks like



The interval 2.9A to 3 A = 0.1 A is very small $\Rightarrow$ can approximate

$P = |\psi(x)|^2\,dx$ with $dx = 0.1\,\text{Å}$ and $x = 2.9\,\text{Å}$ or $3\,\text{Å}$ or $2.95\,\text{Å}$

$\psi(x) = \sqrt{\dfrac{2}{L}}\sin\dfrac{\pi}{L}x$; $|\psi(x)|^2 = \dfrac{2}{L}\sin^2\dfrac{\pi}{L}x$

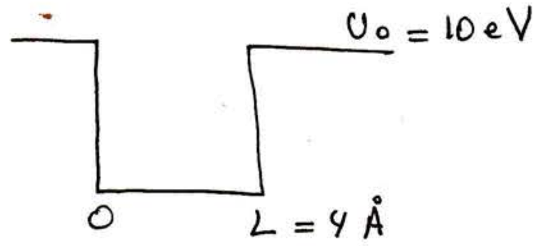
$x = 3\,\text{Å} = \dfrac{3}{4}L \Rightarrow \dfrac{\pi}{L}x = \dfrac{\pi}{L}\cdot\dfrac{3}{4}L = \dfrac{3\pi}{4}$, $\sin\dfrac{3}{4}\pi = \sin\dfrac{\pi}{4} = \dfrac{\sqrt{2}}{2} = \dfrac{1}{\sqrt{2}}$

$\Rightarrow P = \dfrac{2}{4\,\text{Å}}\cdot\left(\dfrac{1}{\sqrt{2}}\right)^2\cdot 0.1\,\text{Å} = \dfrac{2}{4}\cdot\dfrac{1}{2}\cdot 0.1 = \dfrac{0.1}{4} \Rightarrow$

$\boxed{P = \dfrac{0.1}{4} = 0.025}$

The classical probability is $\dfrac{dx}{L} = \dfrac{0.1}{4} \Rightarrow$ $\boxed{\text{is the same}}$

Problem 3



$-\frac{\hbar^2}{2m_e}\Psi'' + U(x)\Psi = E\Psi$; in the regim $x > L$, $U(x) = U_0 \Rightarrow$

$-\frac{\hbar^2}{2m_e}\Psi'' = (E - U_0)\Psi \Rightarrow \quad \Psi'' = \frac{2m_e}{\hbar^2}(U_0 - E)\Psi \equiv \alpha^2 \Psi(x)$

$\Rightarrow \quad \Psi = C e^{-\alpha x} = C e^{-x/\delta}$

$\alpha = \sqrt{\frac{2m_e}{\hbar^2}(U_0 - E)}$, $\delta = \frac{1}{\alpha} = \sqrt{\frac{\hbar^2}{2m_e(U_0 - E)}} = \sqrt{\frac{1973^2}{2\cdot 0.511\cdot 10^6(10 - 2.35)}}\ Å$

$\Rightarrow \boxed{\delta = 0.706\ Å}$ (used $E = 2.35\,eV$)

(b) Assume $L_{eff} = L + 2\delta = 5.41\ Å$

$E_1 = \frac{\hbar^2\pi^2}{2m_e L_{eff}^2} = E_1(\text{box})\cdot\frac{L^2}{L_{eff}^2} = 2.35\,eV \cdot \frac{4^2}{5.41^2}$

$\Rightarrow \boxed{E_1 = 1.28\ eV}$

(c) In the regim $L < x < \infty$, $\Psi(x) = Ce^{-x/\delta} \Rightarrow$

$P(L < x < \delta) = \int_L^\infty dx\,|\Psi(x)|^2 = C^2\int_L^\infty dx\, e^{-2x/\delta} = \boxed{C^2\cdot\frac{\delta}{2}\cdot e^{-2L/\delta}}$

Since $\Psi(x = L) = Ce^{-L/\delta} \Rightarrow \boxed{P(L < x < \delta) = \frac{\delta}{2}\cdot\Psi(x = L)}$

If $\Psi(x = L) = 0.1\ Å^{-1/2} \Rightarrow \boxed{P(L < x < \delta) = 0.0035}$