Formulas:

Time dilation; Length contraction: $\Delta t = \gamma \Delta t' \equiv \gamma\, \Delta t_p$; $L = L_p / \gamma$; $c = 3 \times 10^8 m/s$

Lorentz transformation:

$$x' = \gamma(x - vt) \qquad\qquad x = \gamma(x' + vt')$$

$$y' = y, \quad z' = z \qquad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \qquad y = y', \quad z = z'$$

$$t' = \gamma(t - vx/c^2) \qquad\qquad t = \gamma(t' + vx'/c^2)$$

Spacetime interval: $(\Delta s)^2 = (c\Delta t)^2 - [\Delta x^2 + \Delta y^2 + \Delta z^2]$

Velocity transformation :

$$u_x' = \frac{u_x - v}{1 - u_x v/c^2} \qquad\qquad u_x = \frac{u_x' + v}{1 + u_x' v/c^2}$$

$$u_y' = \frac{u_y}{\gamma(1 - u_x v/c^2)} \qquad\qquad u_y = \frac{u_y'}{\gamma(1 + u_x' v/c^2)}$$

Relativistic Doppler shift: $f_{obs} = f_{source}\sqrt{1 + v/c}\ /\ \sqrt{1 - v/c}$

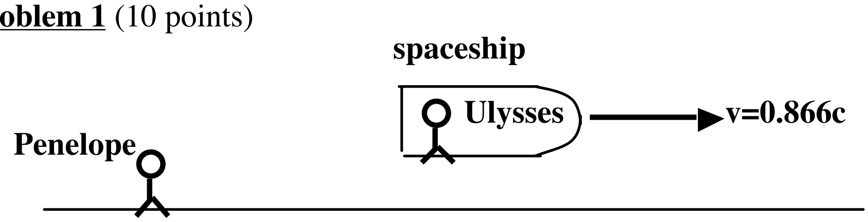
Momentum: $\vec{p} = \gamma m \vec{u}$; Energy: $E = \gamma mc^2$; Kinetic energy: $K = (\gamma - 1)mc^2$

Rest energy: $E_0 = mc^2$; $E = \sqrt{p^2c^2 + m^2c^4}$

Electron: $m_e = 0.511\, MeV/c^2$ Proton: $m_p = 938.26\, MeV/c^2$ Neutron: $m_n = 939.55\, MeV/c^2$

Justify all your answers to all problems. Write clearly.

Problem 1 (10 points)



Ulysses departs on a spaceship going at speed 0.866c, leaving Penelope behind. After one year of travel (according to his clock), he lights up a candle to mark the occasion.

(a) How much time has passed for Penelope since Ulysses departed when Ulysses lights up the candle, as shown on earth's clocks? Solve using the Lorentz transformation formula $t = \gamma(t' + vx'/c^2)$, where the 'primed' frame is the ship.

(b) Answer the same question asked in (a) by using instead the formula $t' = \gamma(t - vx/c^2)$ ('primed' frame is still the ship). Hint: you need to replace x, the position of the ship at time t, then solve for t. Explain why the answer you find is the same as, or different from, the answer found in (a).

(c) When the light from Ulysses' candle finally reaches Penelope, she is overjoyed. How long did she have to wait for this moment since Ulysses departed?

Note: assume the effects of accelerating the spaceship from rest to speed 0.866c can be neglected, so you can deal with inertial frames only.

Problem 2 (10 points)

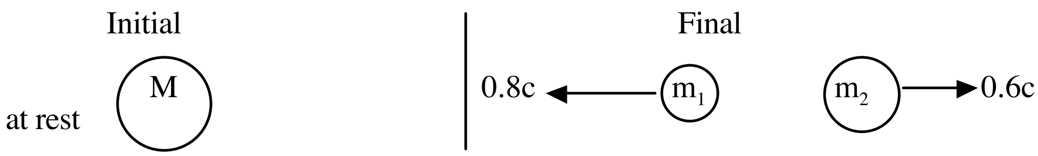
Spaceship A travels to the right at speed 0.6c relative to the ground, spaceship B travels to the left at speed 0.8c relative to the ground. Their length as measured in their own reference frame is 300m.

(a) What is the speed of B as measured from A, and what is the speed of A as measured from B?

(b) What is the length of ship B as measured from the ground?

(c) What is the length of ship B as measured from ship A?

Problem 3 (10 points)



Mass M at rest splits up into two fragments m_1 and m_2 moving at speeds 0.8c and 0.6c respectively as shown in the figure.

(a) Set up the equations for conservation of momentum and energy.

(b) Solve for m_1 and m_2 in terms of M.

(c) Find the kinetic energy of the masses m_1 and m_2, and show that they are accounted for by the mass deficit $\Delta M = M - m_1 - m_2$ that was lost in the disintegration process.