Physics 200B **Mechanics II** **Winter 2013**

Problem Set II : Due Tuesday, February 5, 2013

1.) Derive an expression for the relationship between the unit normal vector to an acoustic path and the profile of the index of refraction. Relate this result to its counterpart for particle motion.

2a.) What are the general conditions on the potential $\mathrm{V}(r,\phi,z)$ for separability of the Hamilton-Jacobi equation for particle motion in cylindrical coordinates?

b.) Solve the Hamilton-Jacobi equation by separation, assuming $\mathrm{V}(r,\phi,z)$ has the requisite form.

3a.) Consider the linearized acoustic equations

$$\frac{\partial \tilde{p}}{\partial t} + \underline{\mathrm{v}} \cdot \nabla \tilde{\rho} = -\rho_0 \underline{\nabla} \cdot \underline{\tilde{\mathrm{v}}}$$

$$\rho_0 \left(\frac{\partial \underline{\tilde{\mathrm{v}}}}{\partial t} + \underline{\mathrm{v}} \cdot \nabla \underline{\tilde{\mathrm{v}}} \right) = -c_s^2 \nabla \tilde{\rho}$$

for an acoustic wave in a *flowing* medium, with flow velocity $\underline{\mathrm{v}}$. This means that the frequency is Doppler shifted. Neglecting the spatial variation of the ambient flow in comparison to the wave length, derive the eikonal equation $\left(\frac{\partial \phi}{\partial t} + \underline{\mathrm{v}} \cdot \underline{\nabla} \phi \right)^2 = (\underline{\nabla} \phi)^2 c_s^2(\underline{x})$.

Show this is equivalent to the predictable:

$$(\omega - \underline{k} \cdot \underline{\mathrm{v}})^2 = k^2 c_s^2(\underline{x}).$$

b.) Now, derive the ray equations with flow.

c.) Explain the physics of the ray equation $\frac{d\underline{k}}{\partial t} = -\frac{\partial}{\partial x}(\omega + \underline{k} \cdot \underline{\mathrm{v}})$. Use this to explain how a vertically sheared horizontal affects acoustic propagation.

d.) Show that the ray equations with flow are Hamiltonian.

4.) Fetter and Walecka 6.6. What are the physical meanings of the "new" coordinates P, Q?

5.) Fetter and Walecka 6.7, 6.8 (counts as one problem)

6.) Fetter and Walecka 6.13

7.) Fetter and Walecka 6.17

8a.) Calculate the phase of the semi-classical wave function for a quantum harmonic oscillator in 1D.

b.) Demonstrate explicity *all* the Poisson bracket relationships discussed in class, apart from the Jacobi identity.