UCSD Physics 130b   *Quantum Mechanics*   Midterm

Prof. B. Keating   November 7, 2014

You are allowed to use two 8.5" x 11" sheets of notes, front and back, which **must** be handwritten. Please write your answers in your blue books & put your name and PID number on each blue book. *Lots of partial credit will be given but **you must write neatly** to receive maximum credit!*

Problem 1: Spin

(a) How many components does the wave function, or "spinor" for a Spin-$\frac{1}{2}$ particle have?

(b) Give an example of a Spin-$\frac{1}{2}$ particle besides an electron.

(c) Apply the spin lowering operator S_- to the ket $\chi_+ = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$ What new ket do you get?

Problem 2: Fine Structure

(a) Write down the two components of the Fine Structure Hamiltonian for the hydrogen atom and briefly explain what causes each them. How big are they compared to the (unperturbed) ground state?

(b) What happens when a large magnetic field is applied to the hydrogen atom? Describe what happens to each of the two components of the Fine Structure perturbation found in part (a).

(c) The Fine Structure effect depends on two quantum numbers: n and j. "Given these corrections to first-order, how many (degenerate) ground states will there be?

(d) What does it mean to "lift" or break the degeneracy, in general (not just for the Fine Structure effect) and how can you, experimentally, "lift" or break this degeneracy in the case of the Fine Structure effect?

Problem 3: Linear perturbation to SHO

In the homework you added cubic and quadratic perturbations to the Simple Harmonic Oscillator. Now, add the easiest perturbation: a linear perturbation with $H' = bx$. Apply the most important equation in quantum mechanics and determine the first-order correction to the ground state energy. Hint: write the perturbation in terms of raising and lowering operators, use Dirac notation for the eigenfunctions of the unperturbed problem (e.g., $|n\rangle$), and take advantage of symmetry.

Problem 4: Delta Square

Consider a particle in the ground state of the infinite square well. Suppose that at time $t = 0$ we suddenly perturb the system by placing a delta function bump of the form $H' = \alpha\delta(x - \frac{a}{2})$ in the middle of the well. Assume that the bump is sufficiently small so that first-order perturbation theory is valid. (a) What exactly does sufficiently small mean here, quantitatively? (b) What is the first order energy of the ground state of the perturbed system? What about the energy of the excited states?