HW 7

1) (a) $M_p = 1.672621777(74) \times 10^{-27}$ kg , pg 4

(b) $Y_0^0 = \frac{1}{\sqrt{4\pi}}$ $\quad Y_1^0 = \left(\frac{3}{4\pi}\right)^{1/2} \cos\theta$ $\quad Y_1^{\pm 1} = \mp\left(\frac{3}{8\pi}\right)^{1/2} \sin\theta \, e^{\pm i\phi}$

$Y_2^0 = \left(\frac{5}{16\pi}\right)^{1/2}(3\cos\theta - 1)$ $\quad Y_2^{\pm 1} = \mp\left(\frac{15}{8\pi}\right)^{1/2} \sin\theta\cos\theta \, e^{\pm i\phi}$

$Y_2^{\pm 2} = \left(\frac{15}{32\pi}\right)^{1/2} \sin^2\theta \, e^{\pm 2i\phi}$ $\quad$ pg. 299

(c) no

(d) $\psi_{n\ell m} = R_{n\ell} Y_\ell^m$

$R_{10} = 2a^{-3/2} e^{-r/a}$ $\quad R_{20} = \frac{1}{\sqrt{2}} a^{-3/2} \frac{r}{a} e^{-r/2a}$

$R_{21} = \frac{1}{\sqrt{24}} a^{-3/2} \frac{r}{a} e^{-r/2a}$

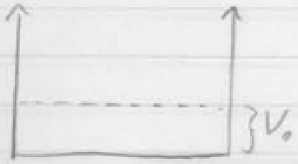
$\underset{1}{R_{10} Y_0^0}, \ \underset{2}{R_{20} Y_0^0}, \ \underset{3}{R_{21} Y_1^1}, \ \underset{4}{R_{21} Y_1^{-1}}, \ \underset{5}{R_{21} Y_1^0}$

(e) $10^{-5} - 10^{-3}$ pb range for spin-independent looks like the best

pg 1501

9.17) start in state N

$$V_0(0) = V_0(T) = 0$$



$$\dot{c}_m = -\frac{i}{\hbar}\sum_n c_n H'_{mn} e^{i(E_m-E_n)t/\hbar}$$

$$H'_{mn} = \langle \psi_m | H' | \psi_n \rangle$$

(a) exact solution

$$H' = V_0 \; , \; H'_{mn} = V_0 \langle \psi_m | \psi_n \rangle = V_0 \delta_{m,n}$$

$$\dot{c}_m = -\frac{i}{\hbar}\sum_n c_n V_0 \delta_{m,n} e^{i(E_m-E_n)t/\hbar} = -\frac{i}{\hbar} V_0 c_m$$

$$\int \frac{dc_m}{c_m} = -\frac{i}{\hbar}\int V_0(t)\,dt$$

$$\boxed{c_m = c_m(0)\exp\left[-\frac{i}{\hbar}\int_0^t V_0(t)\,dt\right]}$$

probability of transition: $|c_m|^2 = |c_m(0)|^2$ no transition

$$c_m = c_m(0)e^{i\phi} \qquad \boxed{\phi = \text{phase shift} = -\frac{1}{\hbar}\int V_0(t)\,dt}$$

(b) 1st order perturbation

$$c_N = 1 - \frac{i}{\hbar}\int_0^t V_0(t)\,dt = 1 + i\phi$$

$$c_m = -\frac{i}{\hbar}\int_0^t H'_{mN} e^{i(E_m-E_n)t/\hbar}\,dt = 0 \qquad b/c \; H'_{mN} = \delta_{mN}$$

this is consistent with the exact solution

$$c_N = c_N(0)e^{i\phi} = e^{i\phi}$$

$$e^{i\phi} = 1 + i\phi + \mathcal{O}(\phi^2) \; ✓$$

2) $\lambda = \frac{1}{\sigma n}$ n = number density $= \rho / m_1$

σ = cross section

m_1 = mass of single proton, found in part (a)

ρ = density of target material

(a) $\rho = 1000\ kg/m^3$: for neutrino : $\lambda = 1.67 \times 10^{17}$

for WIMP : $\lambda = 1.67 \times 10^{10}$

(b) $\rho = 11400$: for neutrino: $\lambda = 1.47 \times 10^{16}$, for WIMP: 1.47×10^{9}

2 (c) XENON-10, b/c it has the lowest upper limit, or COGENT b/c CDMS has the strongest bounds.