PHYSICS 210B : NONEQUILIBRIUM STATISTICAL PHYSICS
HW ASSIGNMENT #3 : STOCHASTIC CALCULUS

(1) Evaluate, for general α, the averages of the following stochastic integrals:

$$\int_0^t dW(s)\, W(s)\, s \qquad , \qquad \int_0^t dW(s)\, W^3(s)\, e^{-\lambda s} \qquad , \qquad \int_0^t dW(s)\, W^{2k+1}(s) \quad .$$

(2) Derive Eqn. 3.99 of the lecture notes.

(3) For the colored noise example in §3.5.3 of the notes, compute numerically $\hat{Y}(\omega)$ and plot your results as a function of $\omega - \nu$. Set $\lambda \equiv 1$ and plot your results for a representative set of different values of the parameter β.

(4) Consider the following stochastic differential equation,

$$dx = -\beta x\, dt + \sqrt{2\beta(a^2 - x^2)}\, dW(t) \ ,$$

where $x \in [-a, a]$.

(i) Find the corresponding Fokker-Planck equation.

(ii) Find the normalized steady state probability $\mathcal{P}(x)$.

(iii) Find and solve for the eigenfunctions $P_n(x)$ and $Q_n(x)$. *Hint: learn a bit about Chebyshev polynomials.*

(iv) Find an expression for $\big\langle x^3(t)\, x^3(0) \big\rangle$, assuming $x_0 \equiv x(0)$ is distributed according to $\mathcal{P}(x_0)$.