PHYSICS 215A: PARTICLES AND FIELDS, FALL 2014
DEPARTMENT OF PHYSICS, UCSD
BENJAMIN GRINSTEIN

Assignment 2

October 22 (Due November 3)

1. Coherent states. Consider the Klein-Gordon theory in 3+1 dimensions.

(i) Show that the state

$$|f\rangle \equiv N \exp\left[\int (dk) f(\vec{k})\alpha^\dagger_{\vec{k}}\right]|0\rangle$$

where $f(\vec{k})$ is some arbitrary function of $\vec{k}$ and N is a normalization constant (that depends on the choice of $f(\vec{k})$), is an eigenvector of the positive frequency (annihilation) part of the Klein-Gordon field, that is, of $\hat{\phi}^{(+)}(\vec{x}) = \int (dk) e^{i\vec{k}\cdot\vec{x}}\alpha_{\vec{k}}$, and determine the eigenvalue. *Hint: First show that* $\alpha_{\vec{k}}|f\rangle = f(\vec{k})|f\rangle$.

(ii) Determine the normalization constant, N, so that $\| \, |f\rangle \, \| = 1$.

(iii) Compute the expectation values of $\hat{H}$, $\hat{\vec{p}}$ and $\hat{N}$ in the state $|f\rangle$. How does $\langle f|\hat{H}|f\rangle$ compare with the energy of the classical field $\phi(x) = \int (dk)\, f(\vec{k})e^{i\vec{k}\cdot\vec{x}}$, and why?

(iv) Show that coherent states stay coherent under time evolution. In particular that $|f_t\rangle \equiv \exp(-iHt)|f\rangle$ is a coherent state with $f_t(\vec{k}) = \exp(-iE_{\vec{k}}t)f(\vec{k})$.

The relativistic invariant measure is $(dk) = d^3k/(2\pi)^3 2E_{\vec{k}}$ and the creation/annihilation operators are normalized relativistically, $[\alpha_{\vec{k}}, \alpha^\dagger_{\vec{k}'}] = (2\pi)^3 2E_{\vec{k}}\delta^{(3)}(\vec{k}-\vec{k}')$.

2. The retarded Green function for the Klein-Gordon equation (in 3+1 dimensions) is given by

$$G_{\text{ret}}(x) = -\int \frac{d^4k}{(2\pi)^4}\frac{e^{-ik\cdot x}}{(k^0+i\epsilon)^2 - \vec{k}^2 - m^2},$$

where the limit $\epsilon \to 0+$ at the end of the calculation is understood.

(i) Show $G_{\text{ret}}(x) = 0$ for $x^2 < 0$, that is, $G_{\text{ret}}(x)$ vanishes outside the light cone of the origin.

(ii) Show by explicit calculation, that for $m = 0$ the explicit form of the Green function is

$$G_{\text{ret}} = \frac{1}{2\pi}\theta(x^0)\delta(x^2).$$

(iii) Compare this with the function $\Delta_+(x)$ introduced in class,

$$\Delta_+(x) = \int (dk)\, e^{-ik\cdot x}.$$

Compute it for $m = 0$.

(iv) Consider the commutator $[\phi(x), \phi(0)]$. Show by explicit computation that it vanishes for space-like separation, $x^2 < 0$.

3. Define the smeared field

$$\phi_f(\vec{x}, t) = \int d^3y\, f(\vec{x} - \vec{y})\phi(\vec{y}, t)\,.$$

Calculate the vacuum expectation value $\langle 0|\phi_f(\vec{x}, t)^2|0\rangle$. Do this for generic f and then explicitly for a Gaussian,

$$f(\vec{x}) = \frac{1}{(2\pi b^2)^{\frac{3}{2}}} e^{-\vec{x}^2/2b^2}\,,$$

in the limit of vanishing mass.