PHYSICS 215A: PARTICLES AND FIELDS, FALL 2014
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Assignment 4

November 17 (Due November 26)
(Will make allowances to turn in on Dec 1, but be advised: new assignment on Nov 26)

1. More about propagators: complex scalar fields.

 (i) If $\phi_1(x)$ and $\phi_2(x)$ are two distinct real "in" scalar fields, show that $\langle 0|T(\phi_1(x)\phi_2(y))|0\rangle = 0$.

 (ii) If $\psi(x)$ is a complex scalar "in" field, determine (the Fourier transform of) $\langle 0|T(\psi(x)\psi(y))|0\rangle$, $\langle 0|T(\psi(x)\psi^\dagger(y))|0\rangle$ and $\langle 0|T(\psi(x)^\dagger\psi^\dagger(y))|0\rangle$. *Hint: Use the result of part (i)*

 (iii) Enunciate a Wick theorem for the time ordered product of complex scalar fields,

$$\langle 0|T(\psi(x_1)\cdots\psi(x_m)\psi^\dagger(y_1)\cdots\psi^\dagger(y_n))|0\rangle.$$

2. Consider a complex scalar field coupled to a complex external (c-number) source,

$$\mathscr{L} = \partial_\mu\psi(x)^*\partial^\mu\psi(x) - m^2\psi(x)^*\psi(x) + \rho(x)\psi(x) + \rho(x)^*\psi(x)^*.$$

Recall that this theory has two types of particles, those of charge "+" and those of charge "-." Assuming the source has compact support (that is, it does not vanish only in some bounded region of space-time, say, a ball around the origin) and that there were no particles at early times, compute the probability of finding, at late times, n_+ particles of type "+" and n_- particles of type "-." By "early" and "late" times we mean, specifically, with respect to the time during which the source is turned on. You may want to first discuss the no particle to no particle probability, and build up the result form there.

3. Consider a real, Klein-Gordon scalar field of mass m coupled to an external source $J(x)$ of compact support (see previous problem for meaning of "compact support"). Assume that at early times there is a one particle state,

$$|g\rangle = \int (dk)\; g(\vec{k})\alpha^\dagger_{\vec{k}}|0\rangle,$$

where $\alpha^\dagger_{\vec{k}}$ are relativistically normalized creation operators, (dk) is the relativistically invariant measure, and $\langle g|g\rangle = 1$.

 (i) Compute the probabilities p_0, $p_{\pm 1}$ that at late times the number of particles has changed by 0, ± 1, respectively.

 (ii) Compare with the result found in class for transition probabilities from a no particles initial state.

(iii) Discuss the role of the source J and the wavepacket g in your result. In particular, what choice of wave-packet makes the creation of an additional particle most probable?

4. *Wick's theorem for normal-ordered products.* In this problem you will find (and use) a useful generalization of Wick's theorem. Unless otherwise stated, all fields are free (*i.e.*, "in").

(i) Expand $T\big(:\phi(x_1)\phi(x_2):\,:\phi(y_1)\phi(y_2):\big)$ as a sum of normal-ordered terms.

(ii) Compare your result with $T\big(\phi(x_1)\phi(x_2)\phi(y_1)\phi(y_2)\big)$. What terms are missing in the expansion of $T\big(:\phi(x_1)\phi(x_2):\,:\phi(y_1)\phi(y_2):\big)$ relative that of $T\big(\phi(x_1)\phi(x_2)\phi(y_1)\phi(y_2)\big)$?

(iii) Use this to give a generalized rule for computing

$$T\big(:\phi(x_1)\cdots\phi(x_k):\,\cdots\,:\phi(x_l)\cdots\phi(x_n):\big).$$