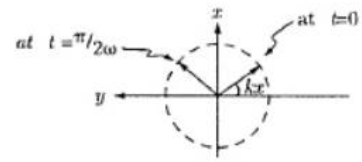
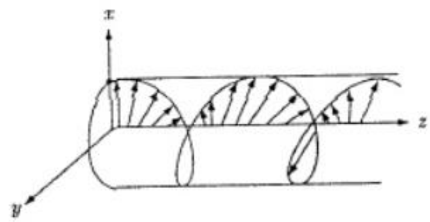
Problem 9.8

(a) $\mathbf{f}_v(z,t) = A\cos(kz-\omega t)\,\hat{\mathbf{x}}$; $\mathbf{f}_h(z,t) = A\cos(kz-\omega t + 90°)\,\hat{\mathbf{y}} = -A\sin(kz-\omega t)\,\hat{\mathbf{y}}$. Since $f_v^2 + f_h^2 = A^2$, the vector sum $\mathbf{f} = \mathbf{f}_v + \mathbf{f}_h$ lies on a circle of radius A. At time $t = 0$, $\mathbf{f} = A\cos(kz)\,\hat{\mathbf{x}} - A\sin(kz)\,\hat{\mathbf{y}}$. At time $t = \pi/2\omega$, $\mathbf{f} = A\cos(kz-90°)\,\hat{\mathbf{x}} - A\sin(kz-90°)\,\hat{\mathbf{y}} = A\sin(kz)\,\hat{\mathbf{x}} + A\cos(kz)\,\hat{\mathbf{y}}$. Evidently it circles **counterclockwise**. To make a wave circling the other way, use $\delta_h = -90°$.



(b)

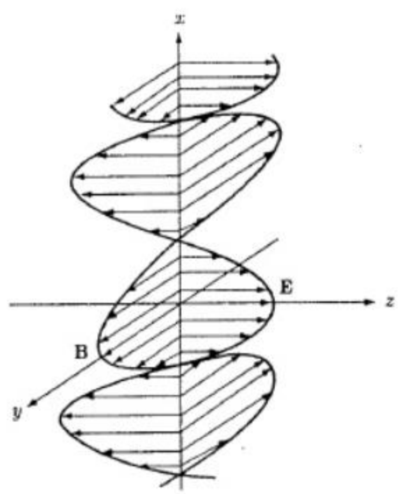


(c) Shake it around in a circle, instead of up and down.

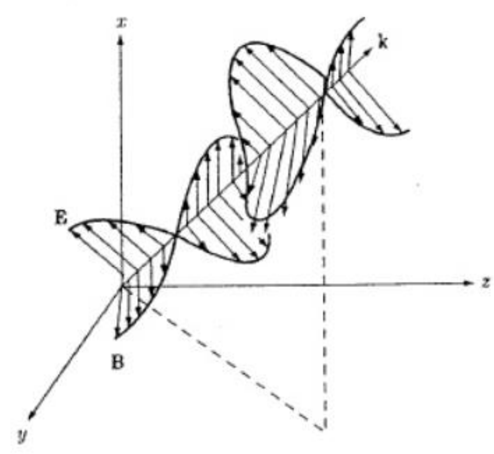
Problem 9.9

(a) $\boxed{\mathbf{k} = -\frac{\omega}{c}\,\hat{\mathbf{x}};\ \hat{\mathbf{n}} = \hat{\mathbf{z}}.}$ $\mathbf{k}\cdot\mathbf{r} = \left(-\frac{\omega}{c}\,\hat{\mathbf{x}}\right)\cdot(x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} + z\,\hat{\mathbf{z}}) = -\frac{\omega}{c}x$; $\mathbf{k}\times\hat{\mathbf{n}} = -\hat{\mathbf{x}}\times\hat{\mathbf{z}} = \hat{\mathbf{y}}$.

$$\boxed{\mathbf{E}(x,t) = E_0\cos\left(\frac{\omega}{c}x + \omega t\right)\hat{\mathbf{z}};\quad \mathbf{B}(x,t) = \frac{E_0}{c}\cos\left(\frac{\omega}{c}x + \omega t\right)\hat{\mathbf{y}}.}$$



(a)



(b)

(b) $\boxed{\mathbf{k} = \frac{\omega}{c}\left(\frac{\hat{\mathbf{x}}+\hat{\mathbf{y}}+\hat{\mathbf{z}}}{\sqrt{3}}\right);\ \hat{\mathbf{n}} = \frac{\hat{\mathbf{x}}-\hat{\mathbf{z}}}{\sqrt{2}}.}$ (Since $\hat{\mathbf{n}}$ is parallel to the xz plane, it must have the form $\alpha\,\hat{\mathbf{x}} + \beta\,\hat{\mathbf{z}}$; since $\hat{\mathbf{n}}\cdot\mathbf{k} = 0, \beta = -\alpha$; and since it is a unit vector, $\alpha = 1/\sqrt{2}$.)

$$\mathbf{k}\cdot\mathbf{r} = \frac{\omega}{\sqrt{3}c}(\hat{\mathbf{x}}+\hat{\mathbf{y}}+\hat{\mathbf{z}})\cdot(x\,\hat{\mathbf{x}}+y\,\hat{\mathbf{y}}+z\,\hat{\mathbf{z}}) = \frac{\omega}{\sqrt{3}c}(x+y+z);\ \hat{\mathbf{k}}\times\hat{\mathbf{n}} = \frac{1}{\sqrt{6}}\begin{vmatrix}\hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}}\\ 1 & 1 & 1\\ 1 & 0 & -1\end{vmatrix} = \frac{1}{\sqrt{6}}(-\hat{\mathbf{x}}+2\,\hat{\mathbf{y}}-\hat{\mathbf{z}}).$$

$$\boxed{\begin{aligned}\mathbf{E}(x,y,z,t) &= E_0\cos\left[\frac{\omega}{\sqrt{3}c}(x+y+z)-\omega t\right]\left(\frac{\hat{\mathbf{x}}-\hat{\mathbf{z}}}{\sqrt{2}}\right);\\ \mathbf{B}(x,y,z,t) &= \frac{E_0}{c}\cos\left[\frac{\omega}{\sqrt{3}c}(x+y+z)-\omega t\right]\left(\frac{-\hat{\mathbf{x}}+2\hat{\mathbf{y}}-\hat{\mathbf{z}}}{\sqrt{6}}\right).\end{aligned}}$$

I currently don't have access to the solutions manual for the 4th edition of the textbook. For this solutions set I used the 3rd edition manual. Problems 9.14 and 9.16 in the 3rd edition correspond to problems 9.15 and 9.17 in the 4th edition respectively.

Problem 9.14

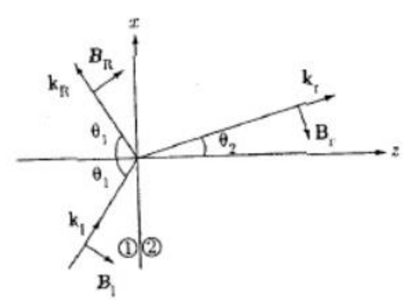
Equation 9.78 is replaced by $\tilde{E}_{0_I}\hat{\mathbf{x}} + \tilde{E}_{0_R}\hat{\mathbf{n}}_R = \tilde{E}_{0_T}\hat{\mathbf{n}}_T$, and Eq. 9.80 becomes $\tilde{E}_{0_I}\hat{\mathbf{y}} - \tilde{E}_{0_R}(\hat{\mathbf{z}}\times\hat{\mathbf{n}}_R) = \beta\tilde{E}_{0_T}(\hat{\mathbf{z}}\times\hat{\mathbf{n}}_T)$. The y component of the first equation is $\tilde{E}_{0_R}\sin\theta_R = \tilde{E}_{0_T}\sin\theta_T$; the x component of the second is $\tilde{E}_{0_R}\sin\theta_R = -\beta\tilde{E}_{0_T}\sin\theta_T$. Comparing these two, we conclude that $\sin\theta_R = \sin\theta_T = 0$, and hence $\theta_R = \theta_T = 0$. qed

Problem 9.16

$$\left\{\begin{aligned}\tilde{\mathbf{E}}_I &= \tilde{E}_{0_I}e^{i(\mathbf{k}_I\cdot\mathbf{r}-\omega t)}\hat{\mathbf{y}},\\ \tilde{\mathbf{B}}_I &= \frac{1}{v_1}\tilde{E}_{0_I}e^{i(\mathbf{k}_I\cdot\mathbf{r}-\omega t)}(-\cos\theta_1\,\hat{\mathbf{x}}+\sin\theta_1\,\hat{\mathbf{z}});\end{aligned}\right\}$$

$$\left\{\begin{aligned}\tilde{\mathbf{E}}_R &= \tilde{E}_{0_R}e^{i(\mathbf{k}_R\cdot\mathbf{r}-\omega t)}\hat{\mathbf{y}},\\ \tilde{\mathbf{B}}_R &= \frac{1}{v_1}\tilde{E}_{0_R}e^{i(\mathbf{k}_R\cdot\mathbf{r}-\omega t)}(\cos\theta_1\,\hat{\mathbf{x}}+\sin\theta_1\,\hat{\mathbf{z}});\end{aligned}\right\}$$

$$\left\{\begin{aligned}\tilde{\mathbf{E}}_T &= \tilde{E}_{0_T}e^{i(\mathbf{k}_T\cdot\mathbf{r}-\omega t)}\hat{\mathbf{y}},\\ \tilde{\mathbf{B}}_T &= \frac{1}{v_2}\tilde{E}_{0_T}e^{i(\mathbf{k}_T\cdot\mathbf{r}-\omega t)}(-\cos\theta_2\,\hat{\mathbf{x}}+\sin\theta_1\,\hat{\mathbf{z}});\end{aligned}\right\}$$



Boundary conditions: $\left\{\begin{array}{ll}(i)\ \epsilon_1 E_1^\perp = \epsilon_2 E_2^\perp, & (iii)\ \mathbf{E}_1^\parallel = \mathbf{E}_2^\parallel,\\ (ii)\ B_1^\perp = B_2^\perp, & (iv)\ \frac{1}{\mu_1}\mathbf{B}_1^\parallel = \frac{1}{\mu_2}\mathbf{B}_2^\parallel.\end{array}\right.$

Law of refraction: $\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1}$. [Note: $\mathbf{k}_I\cdot\mathbf{r}-\omega t = \mathbf{k}_R\cdot\mathbf{r}-\omega t = \mathbf{k}_T\cdot\mathbf{r}-\omega t$, at $z=0$, so we can drop all exponential factors in applying the boundary conditions.]

Boundary condition (i): $0=0$ (trivial). Boundary condition (iii): $\boxed{\tilde{E}_{0_I}+\tilde{E}_{0_R} = \tilde{E}_{0_T}.}$

Boundary condition (ii): $\frac{1}{v_1}\tilde{E}_{0_I}\sin\theta_1 + \frac{1}{v_1}\tilde{E}_{0_R}\sin\theta_1 = \frac{1}{v_2}\tilde{E}_{0_T}\sin\theta_2 \Rightarrow \tilde{E}_{0_I}+\tilde{E}_{0_R} = \left(\frac{v_1\sin\theta_2}{v_2\sin\theta_1}\right)\tilde{E}_{0_T}$.

But the term in parentheses is 1, by the law of refraction, so this is the same as (ii).

Boundary condition (iv): $\frac{1}{\mu_1}\left[\frac{1}{v_1}\tilde{E}_{0_I}(-\cos\theta_1) + \frac{1}{v_1}\tilde{E}_{0_R}\cos\theta_1\right] = \frac{1}{\mu_2 v_2}\tilde{E}_{0_T}(-\cos\theta_2) \Rightarrow$

$\tilde{E}_{0_I} - \tilde{E}_{0_R} = \left(\frac{\mu_1 v_1\cos\theta_2}{\mu_2 v_2\cos\theta_1}\right)\tilde{E}_{0_T}$. Let $\boxed{\alpha \equiv \frac{\cos\theta_2}{\cos\theta_1};\ \beta \equiv \frac{\mu_1 v_1}{\mu_2 v_2}.}$ Then $\boxed{\tilde{E}_{0_I} - \tilde{E}_{0_R} = \alpha\beta\tilde{E}_{0_T}.}$

Solving for $\tilde{E}_{0_R}$ and $\tilde{E}_{0_T}$: $2\tilde{E}_{0_I} = (1+\alpha\beta)\tilde{E}_{0_T} \Rightarrow \tilde{E}_{0_T} = \left(\frac{2}{1+\alpha\beta}\right)\tilde{E}_{0_I}$;

$\tilde{E}_{0_R} = \tilde{E}_{0_T} - \tilde{E}_{0_I} = \left(\frac{2}{1+\alpha\beta} - \frac{1+\alpha\beta}{1+\alpha\beta}\right)\tilde{E}_{0_I} \Rightarrow \tilde{E}_{0_R} = \left(\frac{1-\alpha\beta}{1+\alpha\beta}\right)\tilde{E}_{0_I}$.

Since α and β are positive, it follows that $2/(1+\alpha\beta)$ is positive, and hence the *transmitted* wave is *in phase* with the incident wave, and the (real) amplitudes are related by $\boxed{E_{0_T} = \left(\frac{2}{1+\alpha\beta}\right)E_{0_I}.}$ The *reflected* wave is

Is there a Brewster's angle? Well, $E_{0_R} = 0$ would mean that $\alpha\beta = 1$, and hence that

$$\alpha = \frac{\sqrt{1-(v_2/v_1)^2\sin^2\theta}}{\cos\theta} = \frac{1}{\beta} = \frac{\mu_2 v_2}{\mu_1 v_1}, \text{ or } 1 - \left(\frac{v_2}{v_1}\right)^2\sin^2\theta = \left(\frac{\mu_2 v_2}{\mu_1 v_1}\right)^2\cos^2\theta, \text{ so}$$

$1 = \left(\frac{v_2}{v_1}\right)^2\left[\sin^2\theta + (\mu_2/\mu_1)^2\cos^2\theta\right]$. Since $\mu_1 \approx \mu_2$, this means $1 \approx (v_2/v_1)^2$, which is only true for optically indistinguishable media, in which case there is of *course* no reflection—but that would be true at *any* angle, not just at a special "Brewster's angle". [If μ_2 were substantially different from μ_1, and the relative velocities were just right, it *would* be possible to get a Brewster's angle for this case, at

$$\left(\frac{v_1}{v_2}\right)^2 = 1 - \cos^2\theta + \left(\frac{\mu_2}{\mu_1}\right)^2\cos^2\theta \Rightarrow \cos^2\theta = \frac{(v_1/v_2)^2 - 1}{(\mu_2/\mu_1)^2 - 1} = \frac{(\mu_2\epsilon_2/\mu_1\epsilon_1) - 1}{(\mu_2/\mu_1)^2 - 1} = \frac{(\epsilon_2/\epsilon_1) - (\mu_1/\mu_2)}{(\mu_2/\mu_1) - (\mu_1/\mu_2)}.$$

But the media would be very peculiar.]

By the same token, δ_R is either always 0, or always π, for a given interface—it does not switch over as you change θ, the way it does for polarization in the plane of incidence. In particular, if $\beta = 3/2$, then $\alpha\beta > 1$, for

$$\alpha\beta = \frac{\sqrt{2.25 - \sin^2\theta}}{\cos\theta} > 1 \text{ if } 2.25 - \sin^2\theta > \cos^2\theta, \text{ or } 2.25 > \sin^2\theta + \cos^2\theta = 1. \checkmark$$

In general, for $\beta > 1$, $\alpha\beta > 1$, and hence $\delta_R = \pi$. For $\beta < 1$, $\alpha\beta < 1$, and $\delta_R = 0$.

At normal incidence, $\alpha = 1$, so Fresnel's equations reduce to $E_{0_T} = \left(\frac{2}{1+\beta}\right)E_{0_I}$; $E_{0_R} = \left|\frac{1-\beta}{1+\beta}\right|E_{0_I}$, consistent with Eq. 9.82.

Reflection and Transmission coefficients: $\boxed{R = \left(\frac{E_{0_R}}{E_{0_I}}\right)^2 = \left(\frac{1-\alpha\beta}{1+\alpha\beta}\right)^2.}$ Referring to Eq. 9.116,

in phase if $\alpha\beta < 1$ and 180° out of phase if $\alpha\beta < 1$; the (real) amplitudes are related by $\boxed{E_{0_R} = \left|\frac{1-\alpha\beta}{1+\alpha\beta}\right|E_{0_I}.}$

These are the **Fresnel equations** for polarization perpendicular to the plane of incidence.

To construct the graphs, note that $\alpha\beta = \beta\frac{\sqrt{1-\sin^2\theta/\beta^2}}{\cos\theta} = \frac{\sqrt{\beta^2-\sin^2\theta}}{\cos\theta}$, where θ is the angle of incidence, so, for $\beta = 1.5$, $\alpha\beta = \frac{\sqrt{2.25-\sin^2\theta}}{\cos\theta}$.