PHYSICS 161 Prof: Kim Griest
HOMEWORK 5 DUE: Wed May 7, 2014

1. Start from the radial geodesic equation for the Schwarzschild metric and define the effective potential. Then find radii of the circular orbits. Finally find the value of the angular momentum for the smallest stable orbit, and show that this orbit has $r = 3r_S$. Yes we did this in class, but I want you work through it yourself! Try to do without using the class notes, but use them if you get stuck.

2. Use the method of effective potential from class to prove that the larger circular orbit around a black hole is stable, and that the smaller orbit is unstable. To simplify algebra use $l = 2mr_s$ as the angular momentum. The method is described in the class notes, and be sure to explain each step that you are taking.

3. Draw a picture and describe the behavior of a space ship that comes into a black hole from infinity with a value of E exactly equal to the maximum of the effective potential. Describe it both on the effective potential plot, and in terms of the 3-D orbit. Draw a picture of the trajectory. What happens if the value of E is tiny bit larger or smaller than the maximum?

4. Suppose you live near the center of the Milky Way where there is a $4.3 \times 10^6 M_\odot$ black hole. One day you decide you want to see what the future is like, so you decide to use your spacecraft to spend three weeks very near the Schwarzchild radius of that black hole, and want it to be 50 years in the future when you return.
(a) If you use your powerful engines to just hover above the hole, how close (in radial coordinate r) do you have to go?
(b) While hovering above the Black Hole you decide to call your friends at home. You radio home using using a carrier signal of 60 MHz. What frequency do your friends have to tune their receiver to to pick up your signal?
(c) Your friends also send you messages by radio at 60 Mhz. What frequency do you have to tune to to recieve their calls?

5. Anti-electrons (aka positrons) are produced near the surface of neutron stars when material from a binary companion falls on the surface. Most of these positrons annhilate with electrons to produce gamma rays. What are the energy of these gamma rays (in electron volts) at the neutron star surface? (This is not a GR question!). Consider a neutron star of mass $2M_\odot$ and a radius of 10 km. At what energy should a distant observer on Earth expect to see these photons?

6(a). Define the coordinate speed of a particle orbiting a black hole as $v_\phi = r d\phi/dt$. Use the geodesic equations to find the coordinate speed of a particle in the smallest stable orbit around a $8M_\odot$ black hole.
(b) Find the orbital period (in coordinate time t) for this particle; give the answer in seconds.

7. The Einstein radius is the radius of the ring image formed when a bright source is exactly behind a spherically symmetric lens mass. Using the light bending formula for small angles $\Delta\phi = 4GM/bc^2$, and geometry, show that the Einstein ring radius is $R_E = 2[GMx(L-x)/Lc^2]^{1/2}$, where L is the distance to the source, and x is the distance to the lens mass. To get the above answer you must find the radius of the ring in the lens plane, that is, assume the ring of light is at the distance of the lens mass, x. You should also use the "small angle formula" for triangles, i.e. assume x and L and $L - x$ are much greater than b and r_S.

8. Use the formula above for the Einstein Ring radius to get a feel for lensing scales.
(a) For a source star in the LMC (distance 50 kpc) and a 0.7 $M_\odot$ M-dwarf star lens half way to the LMC, find R_E in AU and also the angular size of the Einstein ring θ_E in arcsec. (You may want to use the back of the book for unit conversion factors.)
(b) Find R_E and θ_E for the same system as above but the lens is 200 parsecs from us along the line-of-sight.
(c) Find R_E and θ_E for a source galaxy 1800Mpc away lensed by cluster of galaxies of mass $10^{15} M_\odot$ at a distance of 700 Mpc.
(d) Find R_E and θ_E for a source galaxy 1800Mpc away lensed by a single galaxy of mass $10^{12} M_\odot$ at a distance of 700Mpc.