Physics 281 **Spring 2014**

Problem Set I: Due Friday, May 30, 2014

1.) Consider a slow moving sheet of length ℓ and width w moving edge-on into a fluid at $Re \ll 1$. What is the drag on the sheet? How does it compare to face-on drag?

2.) a.) Use Π-theorem techniques to show that the lift on an airplane in steady flight scales as:

$$F_L \sim C_L \rho_{air} A_{wing} V^2 .$$

Here ρ_{air} is air density, A_w is wing area, V is speed and C_L is lift efficient. What must and might C_L depend on? Why?

b.) Estimate the minimum possible cruising speed for a fully loaded 747. What thrust is required for that?

3.) A stream of air flows over a long plate of length L at constant speed V. The no-slip condition on the plate's surface creates a *boundary layer* (i.e. a thin layer, of thickness w, around the plate where the flow transitions from zero to V. Consider $VL/\nu \gg 1$ but $Vw/\nu \ll 1$. Take $\underline{\nabla} \cdot \underline{v} = 0$. Assume steady state, so

$$\underline{v} \cdot \underline{\nabla}\, \underline{v} - \nu\, \nabla^2 \underline{v} = -\frac{\nabla P}{\rho} .$$

a.) Estimate the thickness of the boundary layer at distance x along the plate. Assume the BL is laminar, and thickens by viscous diffusion.

b.) If the plate has length L and width b, what is the total drag on the plate?

Assume the plate is edge-on to the flow.

4.) a.) Use the Π theorem to determine the scaling of the along-stream pressure gradient in a pipe with flow V, diameter D, density of water ρ_0, viscosity ν.

b.) Derive the high Re result from direct balance. How is drag produced?

c.) Derive the low Re result from direct balance.

5.) Consider k41 turbulence. Show that, contrary to naïve expectation, the continuum model of a fluid becomes *better* as *Re* increases. Hint: Consider a comparison of ℓ_{mfp} and dissipation scale. Take the turbulence to be subsonic $V << C_s$.

6.) Consider a passive scalar with concentration c which is advected by a turbulent k41 flow, with dissipation rate $\in$ and viscosity ν. Thus c obeys:

$$\frac{\partial}{\partial t} c(\underline{x},t) + \underline{\mathrm{v}} \cdot \underline{\nabla}\ \mathrm{c} - D\ \nabla^2 \mathrm{c} = \tilde{s}\ .$$

Here $\tilde{s}$ represents some input of the scalar, at large scales. Take $D = \nu$, at first.

a.) Derive the scalar concentration structure function $\left\langle (\delta C)^2 \right\rangle$ as a function of ℓ. Explain the result.

b.) Now take $D << \nu$ and $D >> \nu$. Qualitatively explain what will happen.

7.) A fluid rotates in a bucket of radius *R*, with constant-angular velocity *w*. Estimate the depth of the resulting vortex.

8.) A student wearing foul smelling perfume enters in a classroom and sits in the center.

a.) Estimate the time it takes for the stench to fill the room if the room is perfectly still, i.e. the class is sleeping soundly.

b.) Now assume the instructor is energetically moving about the room. Estimate the time for which the stench fills the room.

9.) Define a problem of your own on the topic of dimensional analysis as applied to fluid problems.