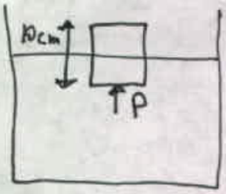
Physics 4B    Quiz 1 Solutions    Spring 2014

Problem 1



The buoyant force $F_B$ equals the weight, since the block floats.
So $F_B = mg$. The pressure acts perpendicular to the surface, so the buoyant force originates in the pressure on the bottom face:

$$F_B = P \cdot A = mg \Rightarrow P = \frac{mg}{A} \quad . \; m = 100\,g = 0.1\,kg, \; A = 100\,cm^2 = 10^{-2}\,m^2$$

$$\Rightarrow P = \frac{0.1\,kg \times 9.8\,m/s^2}{10^{-2}\,m^2} = \boxed{98\,N/m^2} \;(a)$$

(b) The depth is 5 cm, so $P = \rho g h \Rightarrow \rho = \dfrac{P}{gh} \Rightarrow$

$$\Rightarrow \rho = 98\,\frac{N}{m^2}\,\frac{1}{9.8\,\frac{m}{s^2}\,.\,0.05\,m} = \boxed{200\,\frac{kg}{m^3}} \;(b)$$

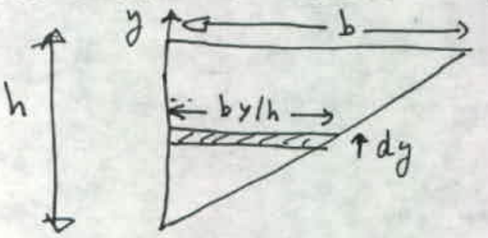
(c) The buoyant force when the block is completely submerged will be $2 \cdot m \cdot g$, so the net force pointing up is $mg$. When it floats, the net force is 0, so the average net force in pushing the block down is $\frac{mg}{2}$, over a distance $d = 5\,cm \Rightarrow$ the work is

$$W = \frac{mg}{2} \times 5\,cm = \frac{0.1\,kg}{2} \times 9.8\,\frac{m}{s^2} \times 0.05\,m \Rightarrow$$

$$\boxed{W = 0.0245\,J}$$

Problem 2

We need to calculate the force for a strip at height $y$, and integrate




The pressure at height $y$ is $P = \rho g(h-y)$

The force on the strip is $dF = P \cdot \frac{by}{h} dy = \rho g(h-y)\frac{by}{h} dy$

So the total force is

$$F = \int_0^h dF = \int_0^h \rho g(h-y)\frac{by}{h}dy = \int_0^h \rho g b y\, dy - \int_0^h \rho g \frac{b}{h} y^2 dy =$$

$$= \rho g b\frac{h^2}{2} - \rho g b\frac{h^2}{3} = \rho g b\frac{h^2}{6}$$

So $\boxed{F = \frac{1}{6}\rho g b h^2}$ (a)

(b) We integrate over the entire rectangular glass + wall now:

$$dF = P.b.dy \Rightarrow dF = \rho g(h-y)\, b\, dy \Rightarrow$$

$$F_{tot} = \int_0^h (\rho g h b - \rho g b y)\, dy = \rho g b\left(h^2 - \frac{h^2}{2}\right) = \boxed{\frac{1}{2}\rho g b h^2}$$

So $\boxed{F_{tot} = \frac{1}{2}\rho g b h^2 = 3F}$

or $\boxed{F = \frac{1}{3}F_{tot}}$

Problem 3

Use Bernoulli law and continuity equation

$$P + \frac{1}{2}\rho v^2 + \rho g y = \text{const.} \quad ; \quad A v = \text{const.} \quad (A = \text{area})$$

For left and middle parts $y$ is the same, for middle and right parts $A$ is the same. $A = \pi d^2/4$, $d$ = diameter

(a) $A_1 v_1 = A_2 v_2 \Rightarrow v_1 = \frac{A_2}{A_1} v_2 = \frac{d_2^2}{d_1^2} v_2 \Rightarrow$

$$v_1 = \frac{6^2}{8^2}\, 8\ \text{m/s} = \frac{36}{8}\ \text{m/s} = \boxed{4.5\ \text{m/s} = v_2}$$

(b) $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2 \Rightarrow$

$$P_2 = P_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) = 50{,}000\ \text{Pa} + \frac{1}{2}\times 1000 \times (4.5^2 - 8^2)\ \text{Pa}$$

$$= 50{,}000\ \text{Pa} - 21{,}875\ \text{Pa} \Rightarrow \boxed{P_2 = 28{,}125\ \text{Pa}}$$

(c) Since $A_2 = A_3$, $\boxed{v_3 = v_2 = 8\ \text{m/s}}$

$$P_3 + \rho g y = P_2 \Rightarrow P_3 = P_2 - \rho g y =$$

$$= 28{,}125\ \text{Pa} - 1000 \times 9.8 \times 0.5\ \text{Pa} =$$

$$= 28{,}125\ \text{Pa} - 4900\ \text{Pa} = \boxed{23{,}225\ \text{Pa} = P_3}$$