Physics 4B Quiz 4 Solutions Spring 2014

Use $g(v_x) = \left(\frac{m}{2\pi k T}\right)^{1/2} e^{-\frac{1}{2}\frac{m v_x^2}{kT}}$

$g(v_x)\,dv_x \cdot N$ gives # of molecules with velocity in x dir. between v_x and $v_x + dv_x$

(a) Velocity distribution is same in all directions. There are twice as many molecules in $dv_y = 4\,m/s$ as in $dv_x = 2\,m/s$ $\Rightarrow$ $\boxed{32,000}$

(b) Let $v_1 = 200\,m/s$, $v_2 = 2v_1 = 400\,m/s$,

$$g(v_x = 0) = \left(\frac{m}{2\pi k T}\right)^{1/2} \; ; \; g(v_2) = \left(\frac{m}{2\pi k T}\right)^{1/2} e^{-\frac{1}{2}\frac{m v_1^2 \cdot 4}{kT}} = g(0)\, e^{-\frac{2 m v_1^2}{kT}}$$

and $g(v_1) = g(0)\, e^{-\frac{1}{2}\frac{m v_1^2}{kT}}$ $\Rightarrow$ $\frac{g(v_2)}{g(0)} = \left(\frac{g(v_1)}{g(0)}\right)^4 = \left(\frac{16,000}{32,000}\right)^4 = \frac{1}{16}$

$\Rightarrow$ there are $32,000/16 = \boxed{2000 \text{ molecules}}$ with $v_x = 400\,m/s \pm 1\,m/s$

(c) $g(0) = 2\,g(v_1) \Rightarrow 1 = 2 e^{-\frac{1}{2}\frac{m v_1^2}{kT}} \Rightarrow \frac{1}{2}\frac{m v_1^2}{kT} = \ln 2 \Rightarrow$

$\Rightarrow T = \frac{1}{2}\frac{m v_1^2}{k \ln 2} = \frac{1}{2} \cdot \frac{32 \cdot 200^2}{8311 \cdot \ln 2}\,K \Rightarrow \boxed{T = 111.1\,K}$

(d) $g(0) = \left(\frac{m}{2\pi k T}\right)^{1/2}$, if T increases by factor of 2 $\Rightarrow$

$\boxed{g(0) \text{ decreases by } 1/\sqrt{2}}$ $\Rightarrow$ 22,627 molecules move with $v_x = 0 \pm 1\,m/s$

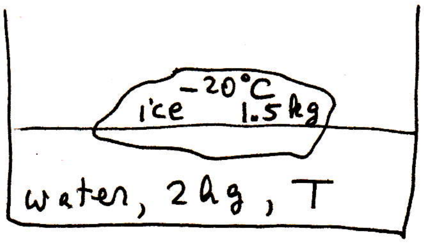
(e) If T increases by factor of 2, $g \to g'$, with

$$g'(v_1) = \left(\frac{m}{2\pi k \cdot 2T}\right)^{1/2} e^{-\frac{1}{2}\frac{m v_1^2}{kT \cdot 2}} = \frac{1}{\sqrt{2}}\left(\frac{m}{2\pi k T}\right)^{1/2}\left(e^{-\frac{1}{2}\frac{m v_1^2}{kT}}\right)^{1/2}$$

Using $e^{-\frac{1}{2}\frac{m v_1^2}{kT}} = \frac{1}{2} \Rightarrow g'(v_1) = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \cdot g(0) = 16,000$

$\Rightarrow$ $\boxed{\text{stays the same}}$

Problem 2



$C_{ice} = 2.1\ kJ/kg\,°C$, $C_{water} = 4.186\ kJ/kg\,°C$, $L_F = 333\ kJ/kg$

(a) Suppose all the water freezes. The heat released is

$$Q = 2kg \times 333\,kJ/kg = 666\ kJ$$

To heat ice from $-20°C$ to $0°C$ requires

$$Q' = 1.5\,kg \times 20°C \times 2.1\,kJ/kg°C = 63\,kJ$$

$\boxed{\text{Since } Q > Q', \text{ not all the water can freeze}}$

(b) Suppose all the ice melts. The heat required is

$$Q_{melt} = 1.5\,kg \times 333\,kJ/kg \doteq 500\,kJ$$

In addition, the heat required for the ice to go to $0°C$ is $63\,kJ$

$\Rightarrow$ $563\,kJ$ = heat required to bring ice to $0°C$ and melt it.

If water cooled from T to 0, heat released is

$$Q_{cool} = 2kg \times 4.186\,kJ/kg°C \times T\,(°C) = 8.372\,T\ kJ/°C$$

$Q_{cool} = 563\,kJ \Rightarrow T = 563/8.372\,°C = 67.2°C$

$\Rightarrow$ $\boxed{\text{for } T > 67.2°C, \text{ all the ice melts}}$

None of the ice will melt if the heat released in bringing the water to $0°C$ is less than the heat needed to bring the ice to $0°C$ $\Rightarrow$

$8.372\,T < 63$ $\Rightarrow$ $\boxed{T < 7.52°C \text{ part of the water freezes}}$

$\Rightarrow$ $\boxed{7.52°C < T < 67.2°C \text{ part of the ice melts}}$

(c) If half the ice melts, heat required is

$$Q_{melt} = 250 \text{ kJ}$$

Everything will be at 0°C. Heat required to heat ice was

63 kJ $\Rightarrow$ total heat required is 313 kJ.

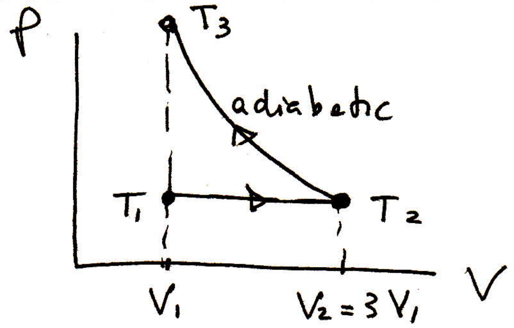
To bring the water to 0°C with 313 kJ, it has to be

at temperature: $8.372 T = 313 \Rightarrow$

$$\Rightarrow \boxed{T = 37.4\,°C}$$

Problem 3

$PV = nRT$



(a) P is constant, $V \to 3V \Rightarrow \boxed{T_2 = 3T_1}$

(b) Work is $W = P\Delta V = P(3V_1 - V_1) = 2PV_1 = 2nRT_1$

Heat absorbed is $Q = C_P \Delta T = C_P(3T_1 - T_1) = 2C_PT_1$

$C_P = \frac{5}{2}nR \Rightarrow Q = 2 \cdot \frac{5}{2}nRT_1 \Rightarrow \boxed{Q = 5nRT_1 = 50{,}000\,J}$

$\Rightarrow \boxed{nRT_1 = 10{,}000\,J} \Rightarrow \boxed{W = 20{,}000\,J}$

$\boxed{\Delta E_{int} = Q - W = 30{,}000\,J \text{ increased}}$

(c) In adiabatic process, $TV^{\gamma-1} = \text{constant} \Rightarrow$

$T_2 V_2^{\gamma-1} = T_3 V_1^{\gamma-1} \Rightarrow 3T_1(3V_1)^{\gamma-1} = T_3 V_1^{\gamma-1} \Rightarrow$

$\Rightarrow T_3 = 3 \cdot 3^{\gamma-1} \cdot T_1$, with $\gamma = \frac{5}{3} \Rightarrow \boxed{T_3 = 3^{5/3}T_1 = 6.24T_1}$

(d) See top of page

(e) $Q = 0 \Rightarrow W = -\Delta E_{int} = -C_V\Delta T = -C_V(6.24T_1 - T_1) =$

$\Rightarrow W = -5.24\,C_V T_1$, using $C_V = \frac{3}{2}nR \Rightarrow$

$W = -5.24 \cdot \frac{3}{2} \cdot nRT_1 = -5.24 \cdot \frac{3}{2} \cdot 10{,}000\,J \Rightarrow$

$\boxed{W = -93.604\,J}$ negative because work was done <u>on</u> the gas