Lecture

Sun is located in Milky Way, or <u>the Galaxy</u>

$\sim 10^{11}$ visible stars

$\sim 10^{10}$ solar masses of gas $\quad 10^{10}\,M_\odot$

$1\,M_\odot = 1.99 \times 10^{33}\,g$

gas has little effect on galactic dynamics

Most of the stars in the Galaxy travel on nearly circular orbits in a thin disk whose radius is of order 10 kiloparsecs and thickness of order 1 kpc

($1\,kpc = 3.086 \times 10^{21}\,cm$)

Typical circular speed: $200\ \frac{km}{s}$

Time required to complete an orbit at 10 kpc is $3 \times 10^8\,yr$

The dispersion in the velocities of stars at a given position is about 40 km/s

Age of the Galaxy: $\sim 10^{10}\,yr = 10\,Gyr$

Typical disk star has completed over 30 revolutions – steady state

Direct collisions are negligible

Kuijken - Dubinski construction

for the bulge DF we take a King model

$$(1) \quad f_{bulge}(E) = \begin{cases} \rho_b (2\pi\sigma_b^2)^{-\frac{3}{2}} \exp\left[(\Psi_0 - \Psi_c)/\sigma_b^2\right]\left\{\exp\left[-(E-\Psi_c)/\sigma_b^2\right]-1\right\} & E < \Psi_c \\ 0 & \text{otherwise} \end{cases}$$

$\psi \leftrightarrow \phi$

three parameters: (1) Ψ_c cutoff potential of bulge

(2) ρ_b approximately central bulge density

(3) σ_b controls velocity dispersion in bulge

Ψ_0 is the gravitational potential at the center of the model

$\rho_{bulge}(\Psi)$ is obtained by integrating its DF over all velocities

$$(2)\quad \rho_{bulge}(\psi) = \rho_b \left[e^{(\psi_0-\psi)/\sigma_b^2} \operatorname{erf}\left(\sqrt{\frac{\psi_c-\psi}{\sigma_b^2}}\right) - \right.$$

$$\left. - \frac{1}{\sqrt{\pi}}\, e^{\frac{\psi_0-\psi_c}{\sigma_b^2}} \left(2\sqrt{\frac{\psi_c-\psi}{\sigma_b^2}} - \frac{4}{3}\left[\frac{\psi_c-\psi}{\sigma_b^2}\right]^{3/2} \right) \right]$$

where $\psi < \psi_c$

Halo (oblate)

$$(3)\quad f_{halo}(E, L_z^2) = \begin{cases} \left[(A \cdot L_z^2 + B)\, e^{-\frac{E}{\sigma_0^2}} + C \right] \left[e^{-\frac{E}{\sigma_0^2}} - 1 \right] & \text{if } E < 0 \\ 0 & \text{otherwise} \end{cases}$$

The density which corresponds to f_{halo} is given by

$$(4)\quad \rho_{halo}(R, \psi) = \frac{1}{2}\pi^{\frac{3}{2}} \sigma_0^3 \left(A \cdot R^2 \sigma_0^2 + 2B \right) \operatorname{erf}\left(\sqrt{-2\psi}/\sigma_0\right) e^{-\frac{2\psi}{\sigma_0^2}}$$

$$+ (2\pi)^{3/2} \sigma_0^3 \left(C - B - A R^2 \sigma_0^2 \right) \operatorname{erf}\left(\sqrt{-\psi}/\sigma_0\right) e^{-\frac{\psi}{\sigma_0^2}}$$

$$+ \pi \sqrt{-2\psi} \left[\sigma_0^2 \left(3A\sigma_0^2 R^2 + 2B - 4C \right) + \frac{4}{3}\psi \left(2C - A\sigma_0^2 R^2 \right) \right]$$

Halo DF has five free parameters:

Ψ_0 potential well depth

σ_0 velocity scale

ρ_1 density scale

R_c core radius

q flattening parameter

$\left.\begin{array}{l}\rho_1 \\ R_c \\ q\end{array}\right\}$ determined by A, B, C

$$R_a = \left(\frac{3}{2\pi G\rho_1}\right)^{\frac{1}{2}} \sigma_0 \, e^{\frac{\Psi_0}{2\sigma_0^2}}$$

characteristic halo radius

↑

can replace ρ_1

$$\Delta\Psi = 4\pi G \rho(\Psi, R)$$

Disk

different vertical and radial dispersions

→ third integral is needed

$$E_z = \psi(R,z) - \psi(R,0) + \frac{1}{2} v_z^2$$

$$f_{disk}(E_p, L_z, E_z) = \frac{\Omega(R_c)}{(2\pi^3)^{\frac{1}{2}} \kappa(R_c)} \cdot \frac{\tilde{\rho}_d(R_c)}{\tilde{\sigma}_R^2(R_c)\tilde{\sigma}_z(R_c)} \times$$

$$\times \exp\left[- \frac{E_p - E_c(R_c)}{\tilde{\sigma}_R^2(R_c)} - \frac{E_z}{\tilde{\sigma}_z^2(R_c)} \right]$$

$E_p = E - E_z$ energy in planar motion

L_z angular momentum along axis of symmetry

R_c radius of circular orbit with angular momentum L_z

E_c energy of same circular orbit

Ω circular frequency at R_c

κ epicyclic frequency at R_c

Calculation of combined potential:

$$\nabla^2 \Psi(R, z) = 4\pi G \left[\rho_{disc}(R, \Psi, \Psi_z) + \rho_{bulge}(\Psi) + \rho_{halo}(R, \Psi) \right]$$

has to be solved numerically