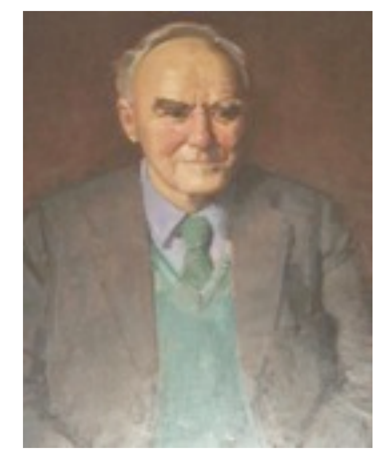
From Reconnection to Relaxation: A Pedagogical Tale of Two Taylors

or: The Physics Assumptions Behind the Color VG

P.H. Diamond



W.C.I. Center for Fusion Theory, N.F.R.I., RoK
and
C.M.T.F.O., U.C.S.D.

Ackn: Y. Kosuga, O.D. Gurcan, T.S. Hahm, L. Wang, Z. Guo, X.G. Wang

This talk focuses on:

- what is the connection between local reconnection and global relaxation?

- how do highly localized reconnection processes, for large Rm, Re, produce global self-organization and structure formation?

We attempt to:

- describe both magnetic fields and flows with similar concepts

- connect and relate to talks by H. Ji, D. Hughes, H. Li, O.D. Gurcan...

- describe self-organization principles

Outline

i.) Preamble: → From Reconnection to Relaxation and Self-Organization

→ What 'Self-Organization' means

→ Why Principles are important

→ Examples of turbulent self-organization

→ Preview

ii.) Focus I: Relaxation in R.F.P. (J.B. Taylor)

→ RFP relaxation, pre-Taylor

→ Taylor Theory
- Summary
- Physics of helicity constraint + hypothesis
- Outcome and Shortcomings

→ Dynamics → Mean Field Theory
- Theoretical Perspective
- Pinch's Perspective
- Some open issues

→ Lessons Learned and Unanswered Questions

Outline

iii.) Focus II: PV Transport and Homogenization (G.I. Taylor)

→ Shear Flow Formation by (Flux-Driven) Wave Turbulence

→ PV and its meaning; representative systems

→ Original Idea: G.I. Taylor, Phil. Trans, 1915, 'Eddy Motion in the Atmosphere'

- Eddy Viscosity, PV Transport and Flow Formation
- Application: Rayleigh from PV perspective

→ Relaxation: PV Homogenization (Prandtl, Batchelor, Rhines, Young)

- Basic Ideas
- Proof of PV Homogenization
- Time Scales
- Relation to Flux Expulsion
- Relation to Minimum Enstrophy states

Outline

→ Does PV Homogenize in Zonal Flows?

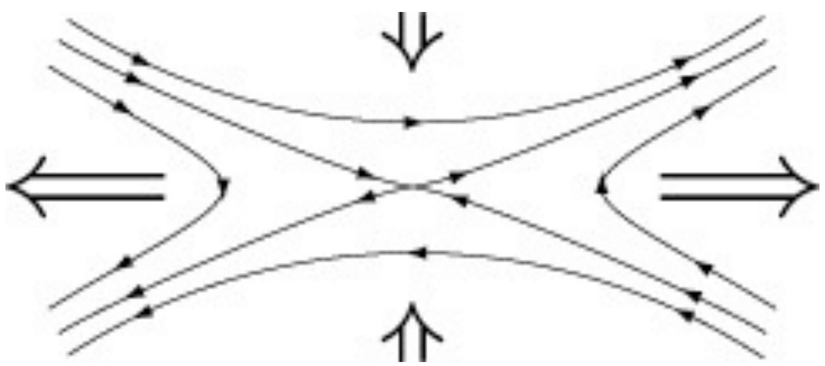
- Physical model and Ideas
- PV Transport and Potential Enstrophy Balance
- Momentum Theorems (Charney-Drazin) and Incomplete Homogenization
- RMP Effects
- $\mathbf{B}_0$ Effects
- Lessons Learned and Unanswered Questions

→ Discussion and General Lessons Learned

I.) Preamble

→ From Reconnection to Relaxation

- Usually envision as localized event involving irreversibility, dissipation etc. at a singularity



S.-P.

$$V = V_A / Rm^{1/2}$$

- ??? - how describe global dynamics of relaxation and self-organization

→

- multiple, interacting/overlapping reconnection events

→ turbulence, stochastic lines, etc

I.) Preamble, cont'd

→ What does 'Self-Organization' mean?

- context: driven, dissipative, open system
- turbulence/stochasticity - multiple reconnection states
- Profile state (resilient, stiff) attractors
- usually, multiple energy channels possible
- bifurcations between attractor states possible
- attractor states macroscopically stable, though may support microturbulence

→ Elements of Theory

- universality (or claims thereof)
- coarse graining - i.e., diffusion
- constraint release - i.e., relaxation of freezing-in law
- selective decay hypothesis

RFP	Tokamak
Taylor/BFM	Stiff core + edge
I_p $\nearrow$ P_{OH} $\searrow$ B profile	Q $\nearrow$ Turbulence $\downarrow\ \uparrow$ Flows
axisymmetric → helical OH	L → H
nearly marginal $m = 1$'s + resistive interchange +...	ITG, CTEM, ... Issue: ELMs?! (domain limited)

- Universality:

Taylor State (Clear)

$$H_M = \int d^3x \mathbf{A} \cdot \mathbf{B}$$

only constraint

Magnetic energy dissipated as H_M conserved

Profile Consistency (especially pedestal) (soft)

PV mixed, subject dynamical constraints

Enstrophy (Turbulence) mixed, dissipated, as macroscopic flow emerges

Why Principles?

→ INSIGHT

→ Physical ideas necessary to guide both physical and digital experiments

→ Principles + Reduced Models required to extract and synthesize lessons from case-by-case analysis

→ Principles guide approach to problem reduction

Examples of Self-Organization Principles

→ Turbulent Pipe Flow: (Prandtl → She)

$$\sigma = -\nu_T \frac{\partial \langle v_y \rangle}{\partial x} \qquad \nu_T \sim v_* x \qquad \Rightarrow \langle v_y \rangle \sim v_* \ln x$$

Streamwise Momentum undergoes scale invariant mixing

→ Magnetic Relaxation: (Woltjer-Taylor)

(RFP, etc)

(Focus 1) Minimize E_M at conserved global H_M $\Rightarrow$ Force-Free RFP profiles

→ PV Homogenization/Minimum Enstrophy: (Taylor, Prandtl, Batchelor, Bretherton, ...)

(Focus 2)

→ PV tends to mix and homogenize

→ Flow structures emergent from selective decay of potential enstrophy relative energy

→ Shakura-Sunyaev Accretion

→ disk accretion enabled by outward viscous angular momentum flux

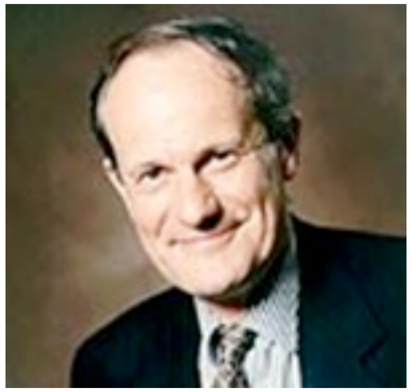
Preview

- Will show many commonalities - though NOT isomorphism - of magnetic and flow self-organization

- Will attempt to expose numerous assumptions in theories thereof

	Magnetic (JB)	Flow (GI)
concept	topology	symmetry
process	turbulent reconnection	PV mixing
players	tearing modes, Alfven waves	drift wave turbulence
mean field	EMF = $\langle \tilde{v} \times \tilde{B} \rangle$	PV Flux = $\langle \tilde{v}_r \tilde{q} \rangle$
constraint	$\int d^3x \mathbf{A} \cdot \mathbf{B}$ conservation	Potential Enstrophy balance
NL	Helicity Density Flux	Pseudomomentum Flux
outcome	B-profiles	zonal flow

II.) Focus I - Magnetic Relaxation



(Derek C Robinson)

→ Prototype of RFP's: Zeta (UK: late 50's - early 60's)

- toroidal pinch = vessel + gas + transformer
- initial results → violent macro-instability, short life time
- weak B_T → stabilized pinch ↔ sausage instability eliminated
- $I_p > Ip, crit$ ($\theta > 1+$) → access to "Quiescent Period"

→ Properties of Quiescent Period:

- macrostability - reduced fluctuations
- $\tau_E \sim 1\ msec$ $T_e \sim 150eV$
- $B_T(a) < 0$ → reversal

→ Quiescent Period is origin of RFP

Further Developments

- Fluctuation studies:

turbulence = $m=1$ kink-tearing → tend toward force-free state

resistive interchange, ...

- Force-Free Bessel Function Model

$$B_\theta = B_0 J_1(\mu r) \qquad B_z = B_0 J_0(\mu r)$$

$$\mathbf{J} = \alpha \mathbf{B}$$

observed to correlate well with observed B structure

- L. Woltjer (1958) : Force-Free Fields at constant α

→ follows from minimized E_M at conserved $\int d^3x \mathbf{A} \cdot \mathbf{B}$

- steady, albeit modest, improvement in RFP performance, operational space

→ Needed: Unifying Principle

Theory of Turbulent Relaxation (J.B. Taylor, 1974)

→ hypothesize that relaxed state minimizes magnetic energy subject to constant global magnetic helicity

i.e. profiles follow from: $$\delta\left[\int d^3x\frac{B^2}{8\pi}+\lambda\int d^3x\,\mathbf{A}\cdot\mathbf{B}\right]=0$$

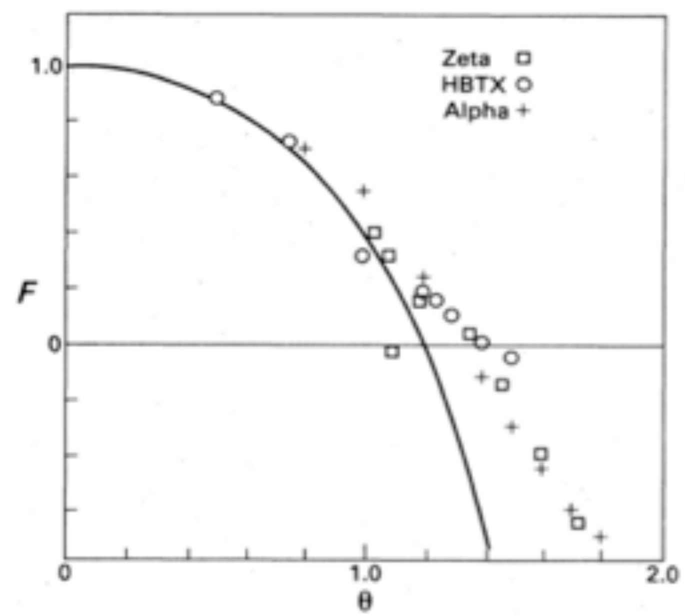
$$\Rightarrow\ \nabla\times\mathbf{B}=\mu\mathbf{B}\ \ ;\ \ J_{\|}/B=\frac{\mathbf{J}\cdot\mathbf{B}}{B^2}=const$$

Taylor state is:

- force free
- flat/homogenized $J_{\|}/B$
- recovers BFM, with reversal for $\theta=\dfrac{2I_p}{aB_0}>1.2$

- Works amazingly well

Result:



$$\theta = \mu a/2 = \frac{2I_p}{aB_0}$$

$$F = B_{z,wall}/\langle B \rangle$$

and numerous other success stories

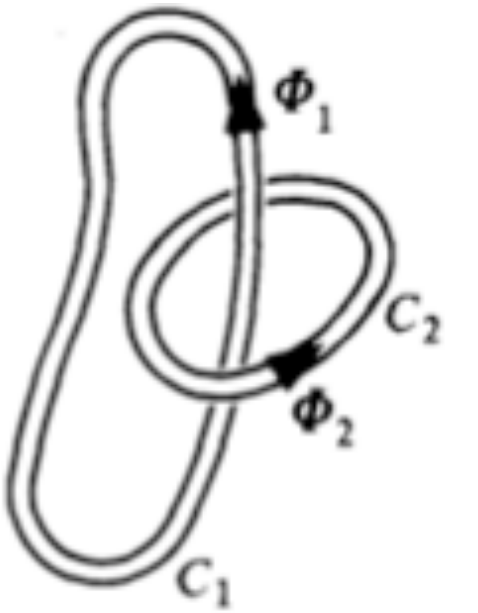
→ Questions:

- what is magnetic helicity and what does it mean?
- why only global magnetic helicity as constraint?
- Theory predicts end state → what can be said about dynamics?
- What does the pinch say about dynamics?

→ Central Issue: Origin of Irreversibility

Magnetic helicity - what is it?

- consider two linked, closed flux tubes



Tube 1: Flux ϕ_1 , contour C_1

Tube 2: Flux ϕ_2 , contour C_2

if consider tube 1: $$H_M^1 = \int_{V_1} d^3x \mathbf{A} \cdot \mathbf{B} = \oint_{C_1} dl \int_{A_1} dS \mathbf{A} \cdot \mathbf{B}$$

$$= \oint_{C_1} d\mathbf{l}_1 \cdot \mathbf{A} \int_{A_1} d\mathbf{a} \cdot \mathbf{B}$$

$$= \phi_1 \oint_{C_1} d\mathbf{l}_1 \cdot \mathbf{A} = \phi_1 \phi_2$$

similarly for tube 2: $H_M^2 = \phi_1 \phi_2$

so $H_M = 2\phi_1\phi_2$

generally : $H_M = \pm 2n\phi_1\phi_2$

- Magnetic helicity measures self-linkage of magnetic configuration

- conserved in ideal MHD - topological invariant

$$\frac{d}{dt} H_M = -2\eta c \int d^3x \mathbf{J} \cdot \mathbf{B}$$

- consequence of Ohm's Law structure, only

N.B.

- can attribute a finite helicity to each closed flux tube with non-constant $q(r)$

- in ideal MHD → ∞ number of tubes in pinch. Can assign infinitesimal tube to each field line

- ∞ number of conserved helicity invariants

  → Follows from freezing in

Question:

How many magnetic field lines in the universe?

(E. Fermi to M.N. Rosenbluth, oral exam at U. Chicago, late 1940's...)

Why Global helicity, Only?

- in ideal plasma, helicity conserved for each line, tube

i.e. $\mathbf{J} = \mu(\alpha, \beta)\mathbf{B} \qquad \mu(\alpha', \beta') \neq \mu(\alpha, \beta)$

- Turbulent mixing eradicates identity of individual flux tubes, lines!

i.e.

- if turbulence s/t field lines stochastic, then '1field line' fills pinch.

1 line ↔ 1tube → only global helicity meaningful.

- in turbulent resistive plasma, reconnection occurs on all scales, but: $\tau_R \sim l^{\alpha} \quad \alpha > 0$

($\alpha = 3/2$ for S-P reconnection)

Thus larger tubes persist longer. Global flux tube most robust

- selective decay: absolute equilibrium stat. mech. suggests possibility of inverse cascade of magnetic helicity (Frisch '75) → large scale helicity most rugged.

Comments and Caveats

→ Taylor's conjecture that global helicity is most rugged invariant remains a conjecture

→ unproven in any rigorous sense

→ many attempts to expand/supplement the Taylor conjecture have had little lasting impact (apologies to some present....)

→ Most plausible argument for global H_M is stochastization of field lines → forces confinement penalty. No free lunch!

→ Bottom Line:

- Taylor theory, simple and successful

- but, no dynamical insight!

Dynamics I:

- The question of Dynamics brings us to mean field theory (c.f. Moffat '78 and an infinity of others - see D. Hughes, Thursday Lecture)

- Mean Field Theory $\rightarrow$ how represent $\langle \tilde{v} \times \tilde{B} \rangle$?

$\rightarrow$ how relate to relaxation?

- Caveat: - MFT assumes fluctuations are small and quasi-Gaussian. They are often NOT

- MFT is often very useful, but often fails miserably

- Structural Approach (Boozer): (plasma frame)

$$\langle \mathbf{E} \rangle = \eta \langle \mathbf{J} \rangle + \langle \mathbf{S} \rangle$$

$\rightarrow$ something $\rightarrow$ related to $\langle \tilde{v} \times \tilde{B} \rangle$

$\langle \mathbf{S} \rangle$ conserves H_M

$\langle \mathbf{S} \rangle$ dissipates E_M

Note this is ad-hoc, forcing $\langle \mathbf{S} \rangle$ to fit the conjecture. Not systematic, in sense of perturbation theory

Now

$$\partial_t H_M = -2c\eta \int d^3x \langle \mathbf{J} \cdot \mathbf{B} \rangle - 2c \int d^3x \langle \mathbf{S} \cdot \mathbf{B} \rangle$$

$$\therefore \langle \mathbf{S} \rangle = \frac{\mathbf{B}}{B^2} \nabla \cdot \mathbf{\Gamma}_H$$

Conservation H_M $\rightarrow$ $\langle S \rangle \sim \nabla \cdot$ (Helicity flux)

$$\partial_t \int d^3x \frac{B^2}{8\pi} = -\int d^3x \left[\eta J^2 - \mathbf{\Gamma}_H \cdot \nabla \frac{\langle \mathbf{J} \rangle \cdot \mathbf{B}}{B^2} \right]$$

so

$\mathbf{\Gamma}_H = -\lambda \nabla (J_{||}/B)$, to dissipate E_M

→ simplest form consistent with Taylor hypothesis

→ turbulent hyper-resistivity $\lambda = \lambda[\langle \tilde{B}^2 \rangle]$ - can derive from QLT

→ Relaxed state: $\nabla (J_{||}/B) \to 0$ homogenized current → flux vanishes

Dynamics II: The Pinch's Perspective

- Boozer model not based on fluctuation structure, dynamics

- Aspects of hyper-resistivity do enter, but so do other effects

→ Point: Dominant fluctuations controlling relaxation are m=1
tearing modes resonant in core → global structure

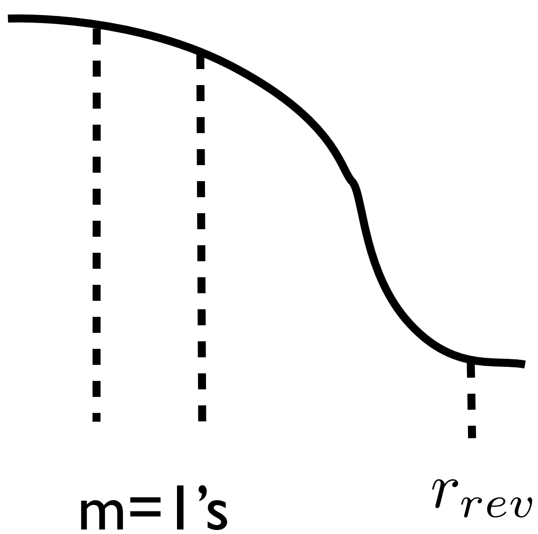
→ Issue: What drives reversal B_z near boundary?

Approach: QL $\langle \tilde{v} \times \tilde{B} \rangle$ in MHD exterior - exercise: derive!

$$\langle \tilde{\mathbf{v}} \times \tilde{\mathbf{B}} \rangle \cong \sum_k |\gamma_k| \frac{R}{r} (q_{res} - q(r)) \langle B_\theta \rangle \partial_r (|\tilde{\xi}_r|_k^2)$$

i.e. $\langle J_\theta \rangle$ driven opposite $\langle B_\theta \rangle$ → drives/sustains reversal

→ What of irreversibility - i.e. how is kink-driven reversal ‘locked-in’?



→ drive $J_{\|}/B$ flattening, so higher n’s destabilized by relaxation front

→ global scattering → propagating reconnection front

m=1, n   m=1, n+1 → m=0, n=1 → driven current sheet, at r_{rev}

(difference beat)

sum beat $\left\{ \begin{array}{l} \text{m=2,} \\ \text{2n+1} \end{array} \right.$

but then m=1, n+2 driven → tearing activity, and relaxation region, broadens

→ Bottom Line: How Pinch ‘Taylors itself’ remains unclear, in detail

Summary of Magnetic Relaxation

concept: topology

process: stochastization of fields, turbulent reconnection

constraint released: local helicity

players: tearing modes

Mean Field: EMF = $\langle \tilde{v} \times \tilde{B} \rangle$

Global Constraint: $\int d^3x \mathbf{A} \cdot \mathbf{B}$

NL: Helicity Density Flux

Outcome: B-Profile

Shortcoming: Rates, confinement $\rightarrow$ turbulent transport

Focus II: Potential Vorticity Mixing $\leftrightarrow$ Iso-vorticity Contour Reconnection

→ Prandtl-Batchelor Theorem and PV Homogenization

→ Self-Organization of Zonal Flows

PV and Its Meaning: Representative Systems

The Fundamentals

- Kelvin's Theorem for rotating system

$$\omega \to \omega + 2\boldsymbol{\Omega}$$

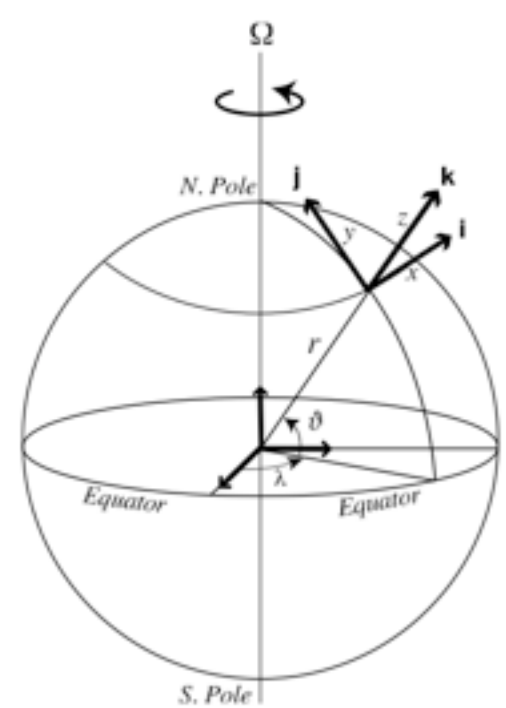
relative   planetary

$$\longrightarrow \quad \oint \mathbf{v} \cdot d\mathbf{l} = \int d\mathbf{a} \cdot (\omega + 2\Omega) \equiv C$$

$\dot{C} = 0$, to viscosity (vortex reconnection)

- $Ro = V/(2\Omega L) \ll 1 \quad \to \quad \mathbf{V} \cong -\nabla_\perp p \times \hat{z}/(2\Omega)$ geostrophic balance

→ 2D dynamics



- Displacement on beta plane

$$\dot{C} = 0 \quad \to \quad \frac{d}{dt}\omega \cong -\frac{2\Omega}{A}\sin\theta_0 \frac{dA}{dt}$$

$$= -2\Omega \frac{d\theta}{dt} = -\beta V_y$$

$\omega = \nabla^2 \phi$, $\quad \beta = 2\Omega \sin\theta_0 / R$

Fundamentals II

- Q.G. equation $$\frac{d}{dt}(\omega + \beta y) = 0$$

n.b. topography

- Locally Conserved PV $q = \omega + \beta y$

$$q = \omega / H + \beta y$$

- Latitudinal displacement → change in relative vorticity

- Linear consequence → Rossby Wave

$$\omega = -\beta k_x / k^2$$

observe: $$v_{g,y} = 2\beta \underline{k_x k_y} / (k^2)^2$$

→ Rossby wave intimately connected to momentum transport

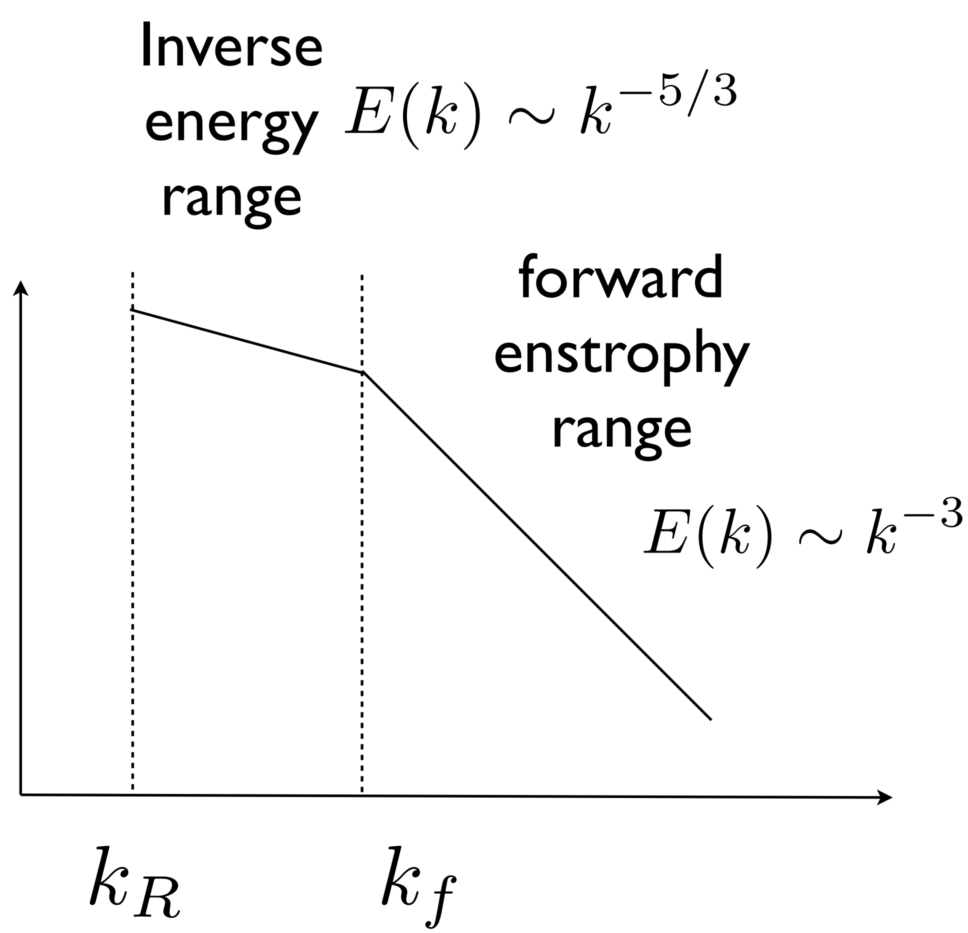
- Latitudinal PV Flux → circulation

- Obligatory re: 2D Fluid

- ω Fundamental: $\partial_t \omega = \nabla \times (\mathbf{V} \times \omega)$

$$\frac{d}{dt}\frac{\omega}{\rho} = \frac{\omega}{\rho} \cdot \nabla \mathbf{V} \quad \rightarrow \text{Stretching}$$

- 2D $\quad d\omega/dt = 0 \quad \rightarrow \quad E = \langle v^2 \rangle, \; \Omega = \langle \omega^2 \rangle$ conserved



How?

$$\partial_t \langle \Delta k^2 \rangle_E > 0 \quad \text{with} \quad \dot{E} = \dot{\Omega} = 0$$

$$\partial_t \langle \Delta k^2 \rangle_E = -\partial_t \bar{k}_E^2$$

dual cascade $\begin{cases} \partial_t \bar{k}_E^{\,2} < 0 & \rightarrow \text{large scale accumulation} \\ \partial_t \bar{k}_\Omega^{\,2} > 0 & \rightarrow \text{flow to small scale dissipation} \end{cases}$

→ Isn't this Meeting about Plasma?

→ 2 Simple Models

a.) Hasegawa-Wakatani (collisional drift inst.)

b.) Hasegawa-Mima (DW)

a.) $\mathbf{V} = \frac{c}{B}\hat{z} \times \nabla\phi + \mathbf{V}_{pol}$

$\sim (\omega/\Omega)$

$L > \lambda_D$ → $\nabla \cdot \mathbf{J} = 0$ → $\nabla_\perp \cdot \mathbf{J}_\perp = -\nabla_\parallel J_\parallel$

$J_\perp = n|e|V_{pol}^{(i)}$

$J_\parallel : \eta J_\parallel = -(1/c)\partial_t A_\parallel - \nabla_\parallel \phi + \nabla_\parallel p_e$

e.s.

b.) $dn_e/dt = 0$

→ $$\frac{dn_e}{dt} + \frac{\nabla_\parallel J_\parallel}{-n_0|e|} = 0$$

n.b.

MHD: $\partial_t A_\parallel$ v.s. $\nabla_\parallel \phi$

DW: $\nabla_\parallel p_e$ v.s. $\nabla_\parallel \phi$

So H-W

$$\rho_s^2 \frac{d}{dt} \nabla^2 \hat{\phi} = -D_\parallel \nabla_\parallel^2 (\hat{\phi} - \hat{n}/n_0) + \nu \nabla^2 \nabla^2 \hat{\phi}$$

$$\frac{d}{dt} n - D_0 \nabla^2 \hat{n} = -D_\parallel \nabla_\parallel^2 (\hat{\phi} - \hat{n}/n_0)$$

$D_\parallel k_\parallel^2/\omega$ is key parameter

n.b. $\mathrm{PV} = n - \rho_s^2 \nabla^2 \phi$ $\qquad \frac{d}{dt}(\mathrm{PV}) = 0$

→ total density

b.) $D_\parallel k_\parallel^2/\omega \gg 1 \to \hat{n}/n_0 \sim e\hat{\phi}/T_e \qquad (m, n \neq 0)$

$$\frac{d}{dt}(\phi - \rho_s^2 \nabla^2 \phi) + v_* \partial_y \phi = 0$$ → H-M

n.b. $\mathrm{PV} = \phi - \rho_s^2 \nabla^2 \phi + \ln n_0(x)$

An infinity of models follow:

- MHD: ideal   ballooning
            resistive $\rightarrow$ RBM

- HW + $A_{\|}$: drift - Alfven

- HW + curv. : drift - RBM

- HM + curv. + Ti: Fluid ITG

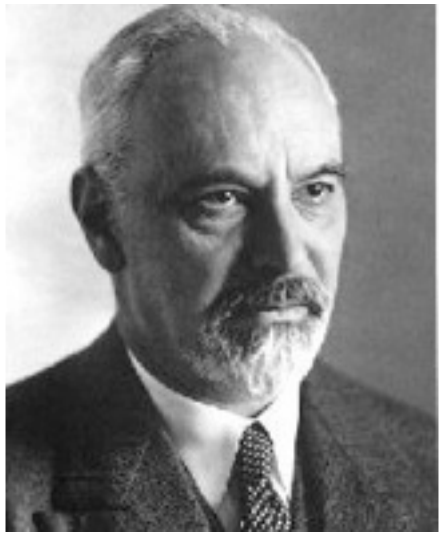
- gyro-fluids

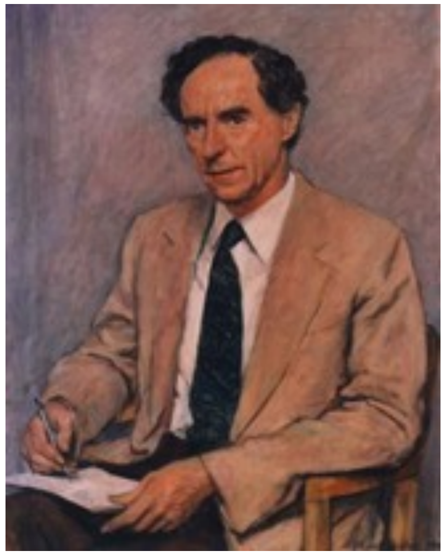
- GK

N.B.: Most Key advances
appeared in consideration
of simplest possible models

Homogenization Theory (Prandtl, Batchelor, Rhines, Young)

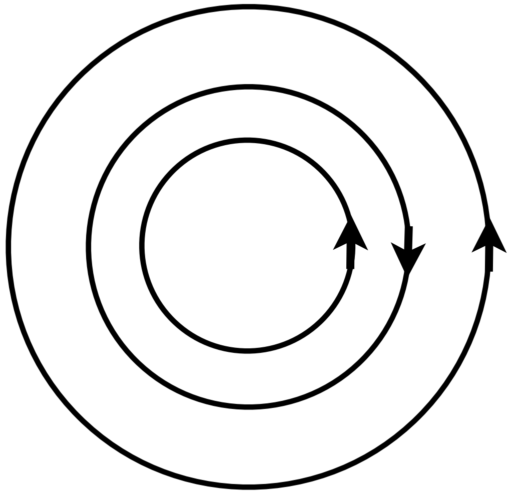
$$\partial_t q + \nabla\phi \times \hat{z} \cdot \nabla q = \nu \nabla^2 q$$





Now: $t \to \infty$ $\partial_t q \to 0$

For $\nu = 0$ $q = q(\phi)$



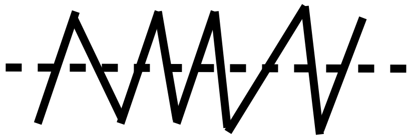
→ $q = q(\phi)$ is arbitrary solution

→ can develop arbitrary fine scale $q = q(\phi)$

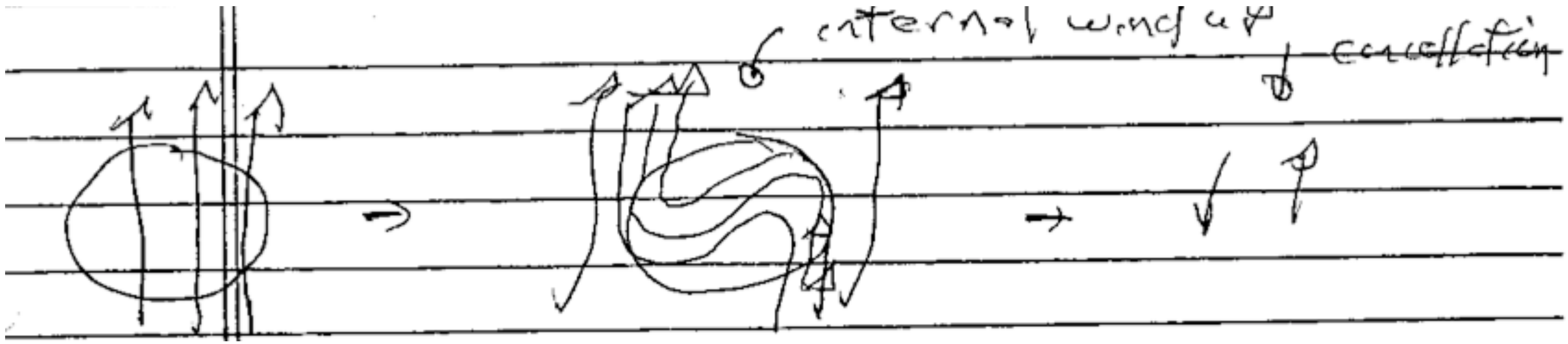
→ closed stream lines, $\nu = 0$

→ no irreversibility

i.e.



Now $\nu \neq 0$



→ non-diffusive stretching produces arbitrary fine scale structure

→ for small, but finite ν , instead of fine scale structure, must have:

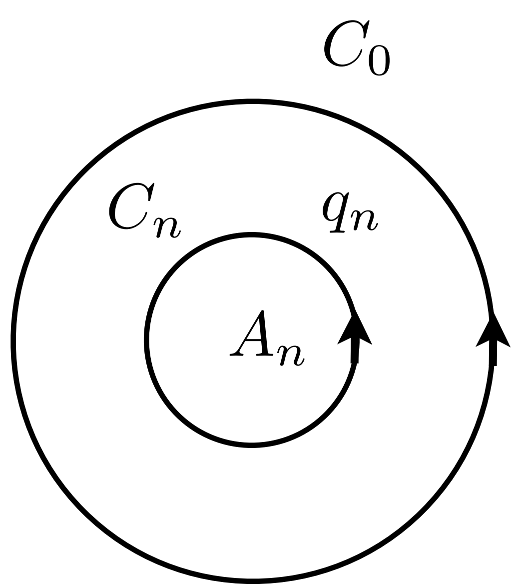
$q(\phi) \to const \qquad t \to \infty$ small ν → global behavior

i.e. finite ν at large Re → PV homogenization

analogy in MHD? →Flux Expulsion

Prandtl - Batchelor Theorem:

Consider a region of 2D incompressible flow (i.e. vorticity advection) enclosed by closed streamline C_0 .
Then, if diffusive dissipation, i.e. $\partial_t q + \nabla\phi \times \hat{z} \cdot \nabla q = \nu \nabla^2 q$
then vorticity → uniform (homogenization), as $t \to \infty$
within C_0



→ underpins notion of PV mixing → basic trend

→ fundamental to selective decay to minimum enstrophy state in 2D fluids (analogue of Taylor hypothesis)

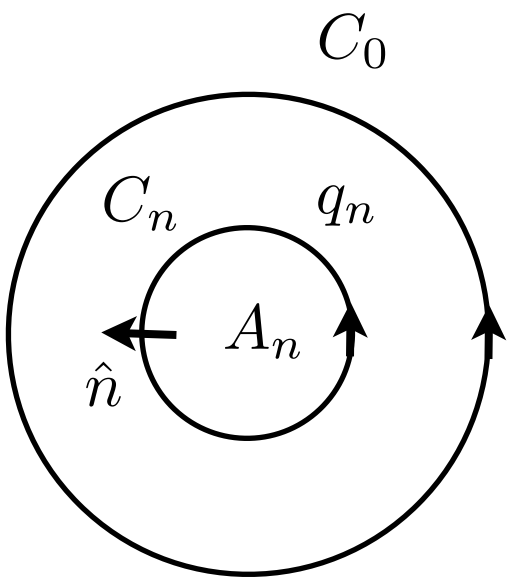
Proof:

$$\int_{A_n} \nabla\cdot(\mathbf{v}q) = 0$$ (closed streamlines)

$$0 = \int_{A_n} \nabla\cdot(\nu\nabla q)$$

$$= \nu\int_{C_n} dl\hat{n}\cdot\nabla q$$

(form of dissipation relevant!)



$C_0 \equiv$ bounding streamline

For $q = q(\phi)$

$$0 = \nu\int_{C_n} dl\hat{n}\cdot\nabla\phi_n \frac{\delta q}{\delta\phi_n}$$

$$= \nu\frac{\delta q}{\delta\phi_n}\int_{C_n} dl\hat{n}\cdot\nabla\phi_n$$

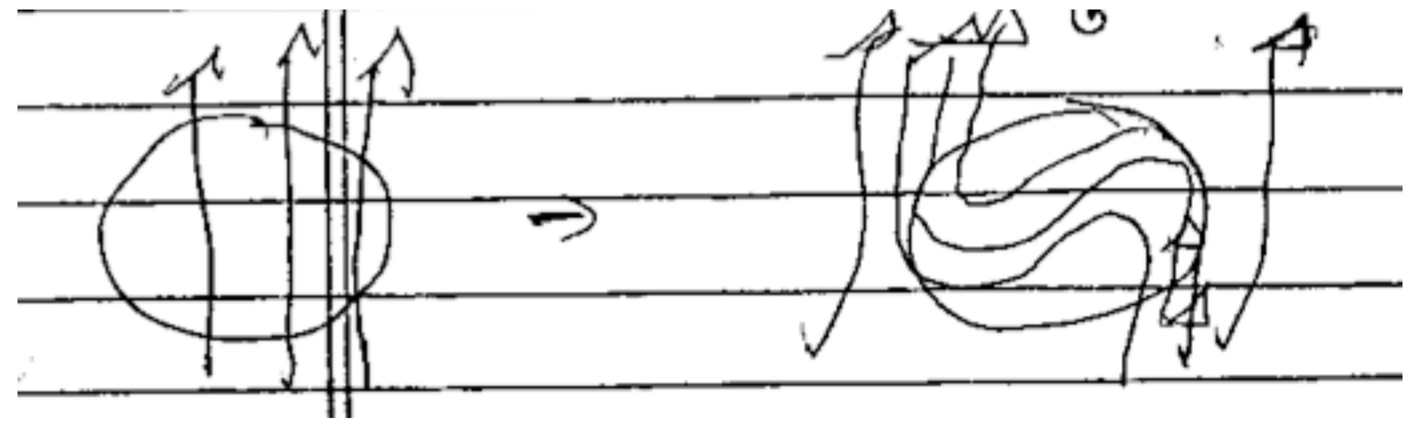
$$\therefore 0 = \nu\frac{\delta q}{\delta\phi_n}\Gamma_n$$

$$\therefore \frac{\delta q}{\delta\phi_n} = 0$$ → q homogenized, within C_0

→ q' tends to flatten!

How long to homogenize? $\longleftrightarrow$ What are the time scales?

Key: Differential Rotation within Eddy



Key: synergism between shear and diffusion

$$1/\tau_{mix} \sim 1/\tau_c (Re)^{-1/3}$$

$\tau_c \equiv$ circulation time

PV homogenization occurs on hybrid decorrelation rate

but $\tau_{mix} \ll \tau_D$ for $Re \gg 1$ $\longrightarrow$ time to homogenize is finite

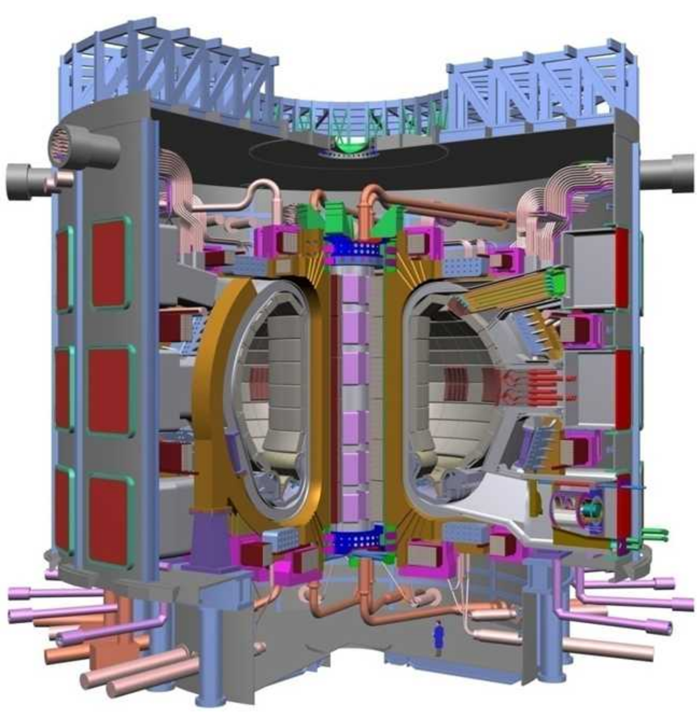
Point of the theorem is global impact of small dissipation - akin Taylor

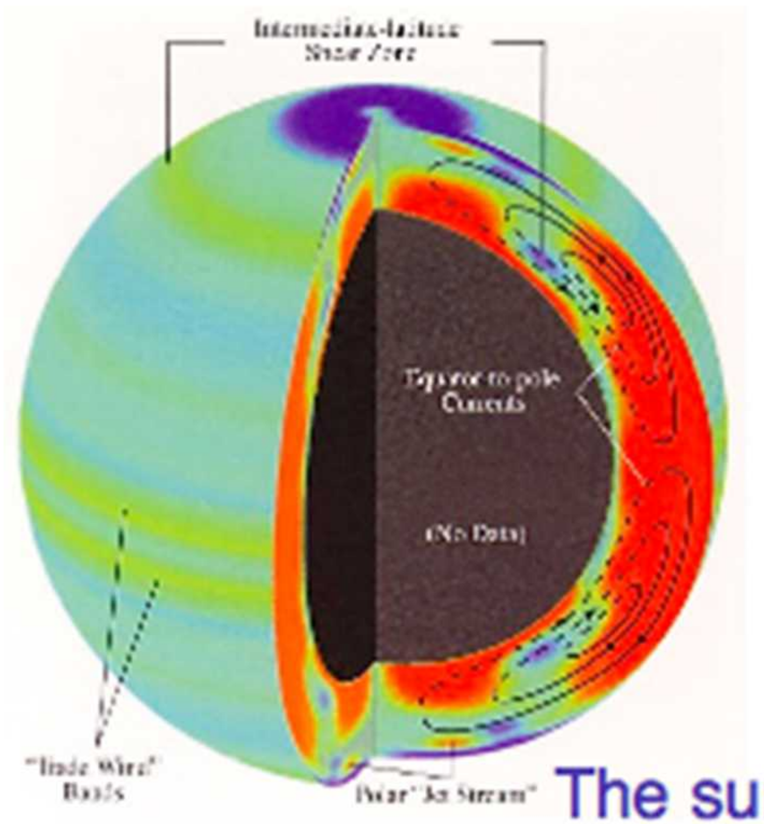
Preamble I

- Zonal Flows Ubiquitous for:
  - ~ 2D fluids / plasmas $R_0 < 1$

    Rotation $\vec{\Omega}$, Magnetization $\vec{B}_0$, Stratification

  Ex: MFE devices, giant planets, stars…





The sun

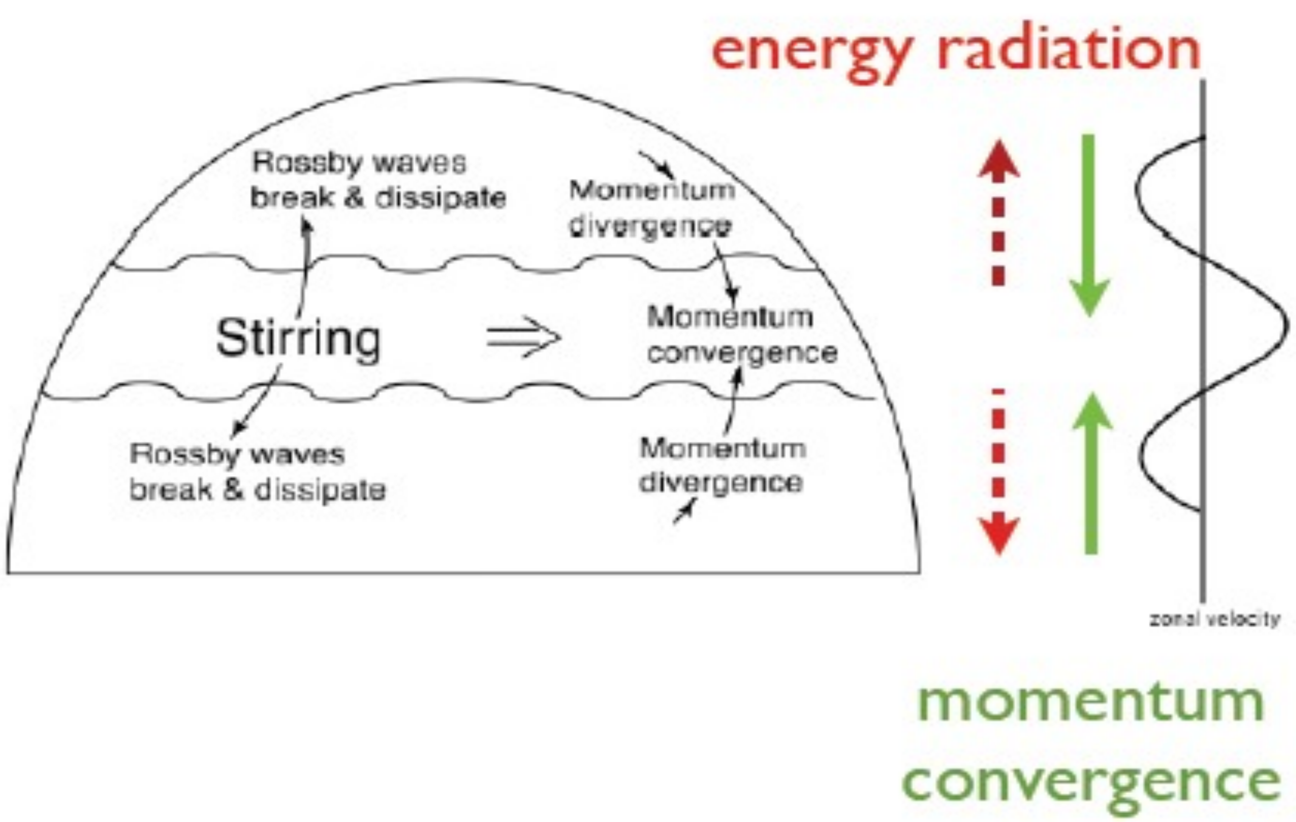
Preamble II

- What is a Zonal Flow?
  - $n = 0$ potential mode; $m = 0$ (ZFZF), with possible sideband (GAM)
  - toroidally, poloidally symmetric *ExB* shear flow
- Why are Z.F.'s important?
  - Zonal flows are secondary (nonlinearly driven):
    - modes of minimal inertia (Hasegawa et. al.; Sagdeev, et. al. '78)
    - modes of minimal damping (Rosenbluth, Hinton '98)
    - drive zero transport ($n = 0$)
  - natural predators to feed off and retain energy released by gradient-driven microturbulence

Heuristics of Zonal Flows a):

Simplest Possible Example: Zonally Averaged Mid-Latitude Circulation

- classic GFD example: Rossby waves + Zonal flow (c.f. Vallis '07, Held '01)
- Key Physics:



Rossby Wave:

$$\omega_k = -\frac{\beta k_x}{k_\perp^2}$$

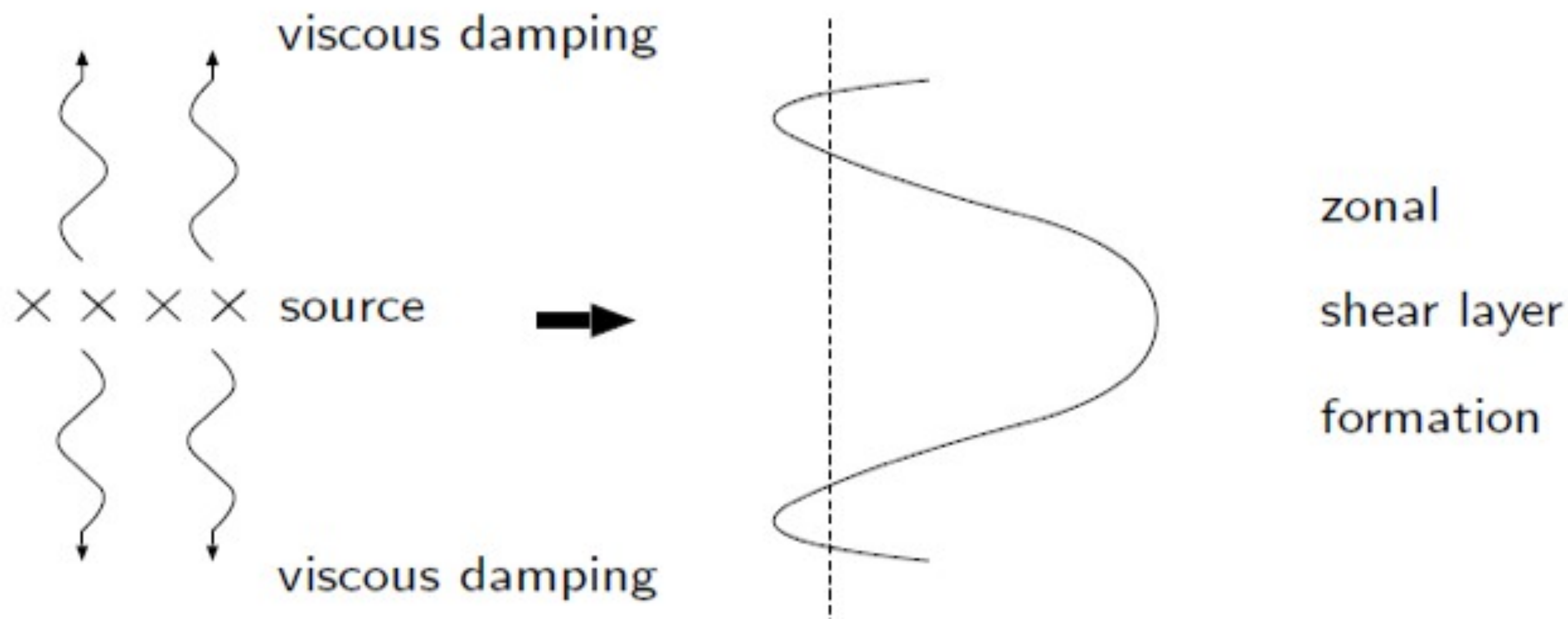
$$v_{gy} = 2\beta\frac{k_x k_y}{k_\perp^2} \qquad \langle \tilde{v}_y \tilde{v}_x \rangle = \sum_k -k_x k_y \left|\hat{\phi}_{\vec{k}}\right|^2$$

$$\therefore \quad v_{gy} v_{phy} < 0$$

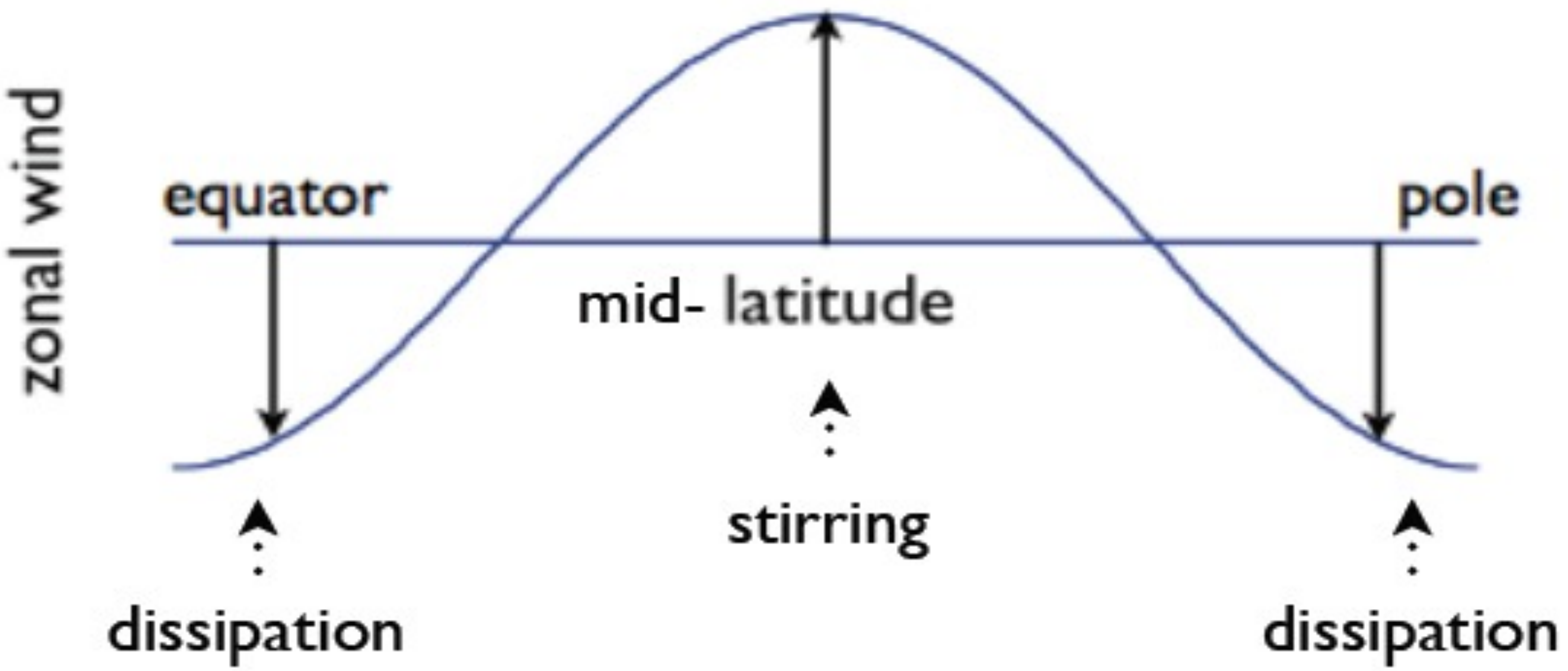
→ Backward wave!

⇒ Momentum convergence at stirring location

- ..."the central result that a rapidly rotating flow, when stirred in a localized region, will converge angular momentum into this region." (I. Held, '01)
- Outgoing waves $\Rightarrow$ incoming wave momentum flux



- Local Flow Direction (northern hemisphere):
  - eastward in source region
  - westward in sink region
  - set by $\beta > 0$
  - Some similarity to spinodal decomposition phenomena
    → both `negative diffusion' phenomena



Key Point: Finite Flow Structure requires *separation* of

excitation and dissipation regions.

=> Spatial structure and wave propagation within are central.

→ momentum transport by waves

Key Elements:

- Waves $\rightarrow$ propagation transports momentum $\leftrightarrow$ stresses

  $\rightarrow$ modest-weak turbulence

- vorticity transport $\rightarrow$ momentum transport $\rightarrow$ Reynolds force

  $\rightarrow$ the Taylor Identity

* - Irreversibility $\rightarrow$ outgoing wave boundary conditions

- symmetry breaking $\rightarrow$ direction, boundary condition

  $\rightarrow \beta$

- Separation of forcing, damping regions

  $\rightarrow$ need damping region broads than source region

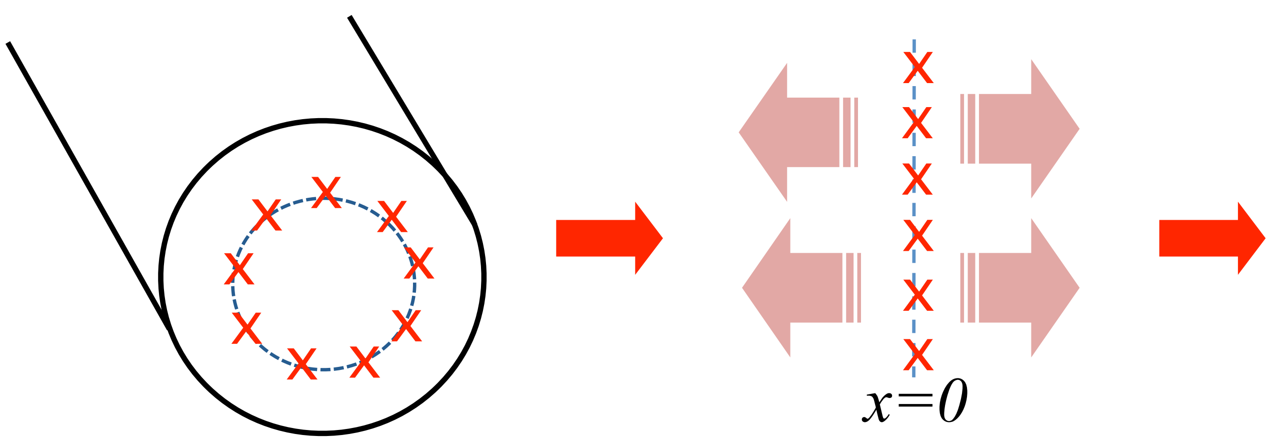
  $\rightarrow$ akin intensity profile...

All have obvious MFE counterparts...

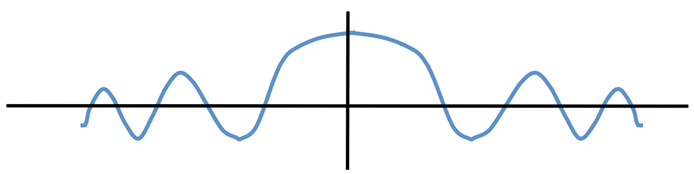
Heuristics of Zonal Flows b.)

2) MFE perspective on Wave Transport in DW Turbulence

- localized source/instability drive intrinsic to drift wave structure



– couple to damping ↔ outgoing wave
i.e. Pearlstein-Berk eigenfunction

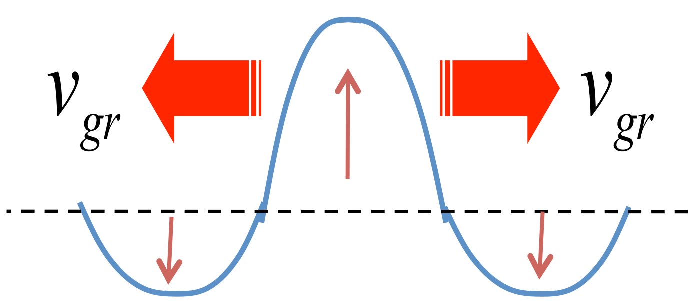


$x > 0 \;\Rightarrow\; v_{gr} > 0$

– $v_{gr} = -2\rho_s^2 \dfrac{k_\theta k_r v_*}{(1+k_\perp^2\rho_s^2)^2}$ $\qquad v_* < 0 \to k_r k_\theta > 0$

– $\langle v_{rE} v_{\theta E}\rangle = -\dfrac{c^2}{B^2}|\phi_{\vec{k}}|^2 \, k_r k_\theta < 0$

- outgoing wave energy flux → incoming wave momentum flux
  → counter flow spin-up!

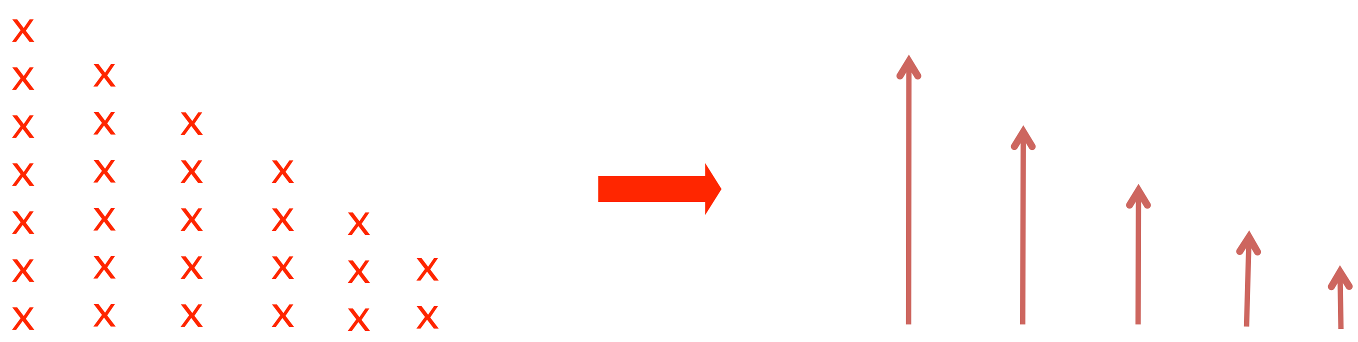


- zonal flow layers form at excitation regions

NFRI National Fusion Research Institute

Heuristics of Zonal Flows b.) cont'd

- So, if spectral intensity gradient → net shear flow → mean shear formation



$$S_r = v_{gr}\varepsilon \cong -\frac{2k_r k_\theta V_t \rho_*^2}{\left(1+k_\perp^2\rho^2\right)^2}\varepsilon$$

$$\langle \tilde{v}_r \tilde{v}_\theta \rangle \approx \sum_k -k_r k_\theta \left|\phi_{\vec{k}}\right|^2$$

- Reynolds stress proportional radial wave energy flux $\vec{S}$, mode propagation physics (Diamond, Kim '91)
- Equivalently: $\partial_t E + \nabla \cdot \mathbf{S} + (\omega \mathrm{Im}\omega) E = 0$ (Wave Energy Theorem)
  - $\therefore$ Wave dissipation coupling sets Reynolds force at stationarity
- Interplay of drift wave and ZF drive originates in mode dielectric
- Generic mechanism…

Towards Calculating Something: Revisiting Rayleigh from PV Perspective

– G.I. Taylor's take on Rayleigh criterion

- consider effect on (zonal) flow by displacement of PV: δy

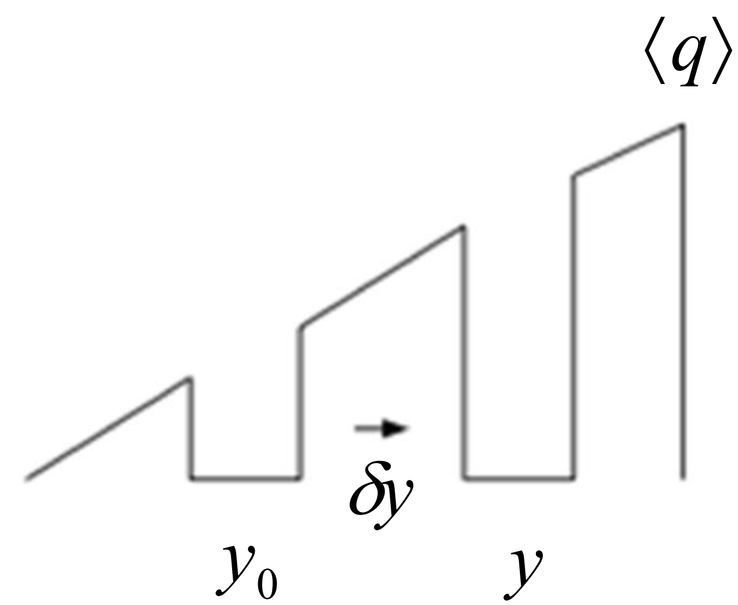
$$\frac{\partial}{\partial t}\langle v_x \rangle = \langle \tilde{v}_y \tilde{q} \rangle$$

$$\tilde{q} = (\text{PV of vorticity blob at y}) - (\text{mean PV at y})$$

$$\langle q(y) \rangle = \langle q(y_0) \rangle + (y - y_0)\left.\frac{d\langle q \rangle}{dy}\right|_{y_0}$$

Small displacement

$$\therefore \frac{\partial}{\partial t}\langle v_x \rangle = -\langle \tilde{v}_y \delta y \rangle \frac{d\langle q \rangle}{dy} = -\left(\partial_t \frac{\langle \tilde{\varepsilon}^2 \rangle}{2} \frac{d\langle q \rangle}{dy} \right)$$



Flow driven by PV Flux

So, for instability $\begin{cases} \partial_t \langle \tilde{\varepsilon}^2 \rangle > 0 & \text{; growing displacement} \\ \dfrac{\partial}{\partial t} \displaystyle\int_{-a}^{a} dy \langle v_x \rangle = 0 & \text{; momentum conservation} \end{cases}$

$$-\int_{-a}^{a} dy \left(\partial_t \frac{\langle \tilde{\varepsilon}^2 \rangle}{2} \right) \frac{d\langle q \rangle}{dy} = 0$$

$\dfrac{d\langle q \rangle}{dy}$ must change sign within flow interval

$\Rightarrow$ inflection point

also,

$$\frac{\partial}{\partial t} \left\{ \langle v_x \rangle + \frac{\langle \tilde{\varepsilon}^2 \rangle}{2} \frac{d\langle q \rangle}{dy} \right\} = 0 \qquad \tilde{q} = -\tilde{\varepsilon} \frac{d\langle q \rangle}{dy}$$

$$\Rightarrow \boxed{\frac{\partial}{\partial t} \left\{ \langle v_x \rangle - \left(-\frac{\langle \tilde{q}^2 \rangle}{2\partial \langle q \rangle / \partial y} \right) \right\} = 0}$$

$-\langle \tilde{q}^2 \rangle / 2\partial \langle q \rangle / \partial y \equiv$ Pseudomomentum for QG system

$\rightarrow$ no slip condition of flow + quasi-particle gas

$\rightarrow$ (significant) step toward momentum theorem

i.e. ties flow to wave momentum density

Zonal Flows I

- Fundamental Idea:
  - Potential vorticity transport + 1 direction of translation symmetry
    → Zonal flow in magnetized plasma / QG fluid
  - Kelvin's theorem is ultimate foundation

- G.C. ambipolarity breaking → polarization charge flux → Reynolds force
  - Polarization charge ➡ $-\boxed{\rho^2}\nabla^2\phi = \boxed{n_{i,GC}(\phi) - n_e(\phi)}$

    polarization length scale — *ion GC* — *electron density*

  - so $\Gamma_{i,GC} \neq \Gamma_e$ ➡ $\rho^2\left\langle \tilde{v}_{rE}\nabla_\perp^2\tilde{\phi}\right\rangle \neq 0$ ⬌ 'PV transport'

    polarization flux → What sets cross-phase?
  - If 1 direction of symmetry (or near symmetry):

    $-\rho^2\left\langle \tilde{v}_{rE}\nabla_\perp^2\tilde{\phi}\right\rangle = -\partial_r\left\langle \tilde{v}_{rE}\tilde{v}_{\perp E}\right\rangle$ (Taylor, 1915)

    $-\partial_r\left\langle \tilde{v}_{rE}\tilde{v}_{\perp E}\right\rangle$ ➡ Reynolds force ➡ Flow

Notable by Absence: Three "Usual Suspects"

- "Inverse Cascade"
  - Wave mechanism is essentially linear

    → scale separation often dubious
  - PV transport is sufficient / fundamental
- "Rhines Mechanism"
  - requires very broad dynamic range
  - Waves ⇔ k_R ⇔ forced strong turbulence
  - strong turbulence model
- "Modulational Instability"

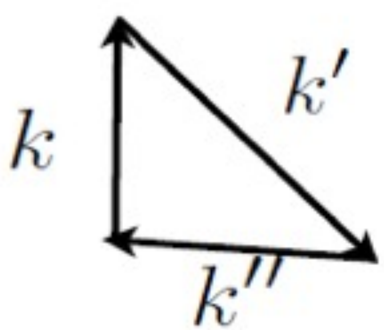
  → see P.D. et al. PPCF'05, CUP'10 for detailed discussion
  - coherent, quasi-coherent wave process
  - useful concept, but not fundamental

Lesson: Formation of zonal bands is generic to the response of a rapidly rotationg fluid to any localized perturbation

Inverse Cascade/Rhines Mechanism

$$k \begin{cases} \omega_k \sim -\beta k_x / k^2 \\ 1/\tau_k \end{cases}$$

transfer <=> triad couplings



eddy transfer: $\omega_{MM} < 1/\tau_c$

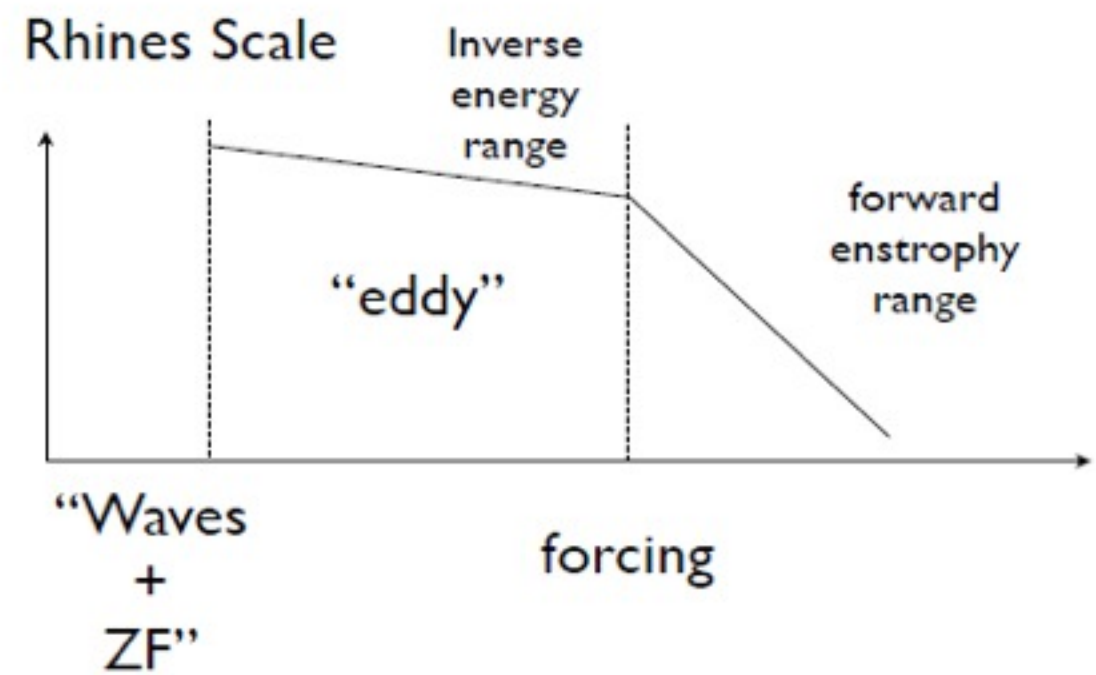
wave transfer: $\omega_{MM} > 1/\tau_c$

cross over: $\omega_{MM} \sim 1/\tau_c$

=> Rhines Scale - emergent characteristic scale for ZF

$$l_R \sim (\tilde{v}/\beta)^{1/2} \sim \epsilon^{1/5}/\beta^{3/5}$$

Contrast: Rhines mechanism vs critical balance



triads: 2 waves + ZF

The crux:

- 3 wave resonance requires 1 wave with $k_x = 0$

- ZF's appear at k_R

- coupling maximal at k_R

=> k_R Z.F. dominates

→ Caveat Emptor:

- often said `Zonal Flow Formation $\cong$ Inverse Cascade'

<u>but</u>

- anisotropy crucial → $\langle \tilde{V}^2 \rangle$, β, forcing → ZF scale

- numerous instances with:
  - no inverse inertial range
  - ZF formation $\leftrightarrow$ quasi-coherent

all really needed:

$$\langle \tilde{V}_y \tilde{q} \rangle \rightarrow \text{PV Flux} \rightarrow \langle \tilde{V}_y \tilde{V}_x \rangle \rightarrow \text{Flow}$$

→ transport and mixing of PV are fundamental elements of dynamics

Zonal Flows II

- Potential vorticity transport and momentum balance
  - Example: Simplest interesting system → Hasegawa-Wakatani
    - Vorticity: $\frac{d}{dt}\nabla^2\phi = -D_{\|}\nabla_{\|}^2(\phi - n) + D_0\nabla^2\nabla^2\phi$
    - Density: $\frac{dn}{dt} = -D_{\|}\nabla_{\|}^2(\phi - n) + D_0\nabla^2 n$

    $\left\{ \begin{array}{l} D_0 \quad \text{classical, feeble} \\ \Pr = 1 \quad \text{for simplicity} \end{array} \right.$
  - Locally advected PV: $q = n - \nabla\phi^2$
    - PV: charge density $\left\{ \begin{array}{l} n \rightarrow \text{guiding centers} \\ -\nabla\phi^2 \rightarrow \text{polarization} \end{array} \right.$
    - conserved on trajectories in inviscid theory $\boxed{dq/dt=0}$
    - PV conservation → $\left.\begin{array}{l}\text{Freezing-in law} \\ \text{Kelvin's theorem}\end{array}\right\} \rightarrow$ Dynamical constraint

Zonal Flow II, cont'd

- Potential Enstrophy (P.E.) balance

$d\langle q^2\rangle / dt = 0$ flux dissipation $\langle\ \rangle \to$ coarse graining

$$\text{LHS} \Rightarrow \frac{d}{dt}\langle\tilde{q}^2\rangle \equiv \partial_t\langle\tilde{q}^2\rangle + \partial_r\langle\tilde{V}_r\tilde{q}^2\rangle + D_0\langle(\nabla\tilde{q})^2\rangle$$

$\text{RHS} \Rightarrow \text{P.E. evolution} -\langle\tilde{V}_r\tilde{q}\rangle\langle q\rangle' \Rightarrow$ P.E. Production by PV mixing / flux

- PV flux: $\langle\tilde{V}_r\tilde{q}\rangle = \langle\tilde{V}_r\tilde{n}\rangle - \langle\tilde{V}_r\nabla^2\tilde{\phi}\rangle$; $\text{but}: \langle\tilde{V}_r\nabla^2\tilde{\phi}\rangle = \partial_r\langle\tilde{V}_r\tilde{V}_\theta\rangle$

$\therefore$ P.E. production directly couples driving transport and flow drive

- Fundamental Stationarity Relation for Vorticity flux

$$\langle\tilde{V}_r\nabla^2\tilde{\phi}\rangle = \langle\tilde{V}_r\tilde{n}\rangle + (\delta_t\langle\tilde{q}^2\rangle)/\langle q\rangle'$$

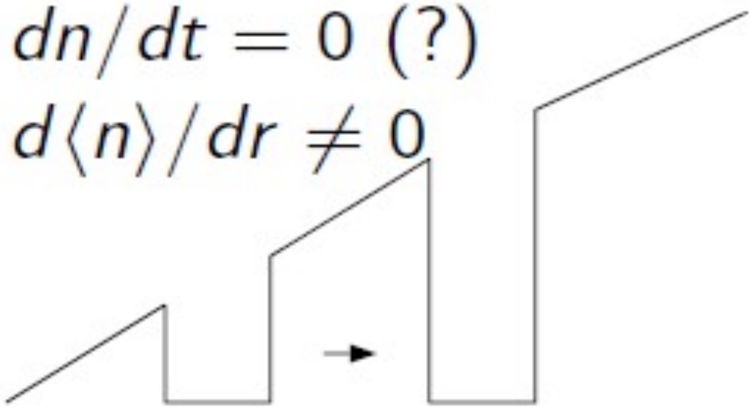
Reynolds force Relaxation Local PE decrement

$\therefore$ Reynolds force locked to driving flux and P.E. decrement; transcends quasilinear theory

Contrast: Implications of PV Freezing-in Law

$dn/dt = 0$ (?)

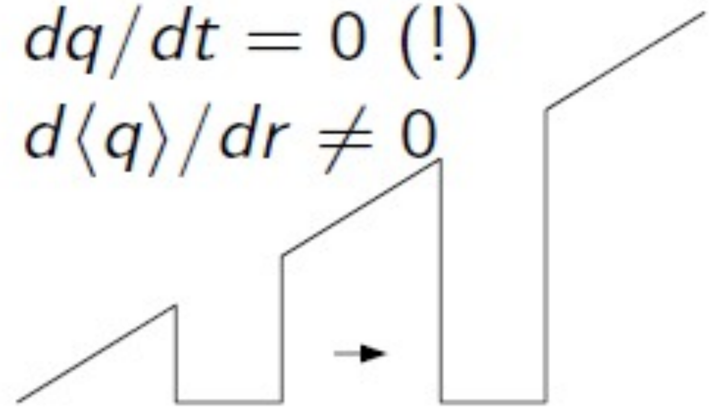
$d\langle n\rangle/dr \neq 0$



$\tilde{n}$ grows $\rightarrow \langle \tilde{V}_r \tilde{n} \rangle \rightarrow$:-(

$dq/dt = 0$ (!)

$d\langle q\rangle/dr \neq 0$



$\tilde{q}$ grows

$$\rightarrow \begin{cases} \langle \tilde{V}_r \tilde{n} \rangle \rightarrow \text{ transport} \rightarrow \text{:-(} \\ \langle \tilde{V}_r \nabla^2 \tilde{\phi} \rangle \rightarrow \text{ flow} \rightarrow \text{:-)} \end{cases}$$

Lesson: Even if $\langle q \rangle \cong \langle n \rangle$, PV conservation must channel free energy into zonal flows!

Key Question: Branching ratio of energy coupled to flow vs transport-inducing fluctuations?

- Combine: $\begin{cases} \text{PE balance} \\ \partial_t \langle V_\theta \rangle = -\langle \tilde{V}_r \nabla^2 \tilde{\phi} \rangle - \nu \langle V_\theta \rangle \end{cases}$ yields...
- Charney-Drazin Momentum Theorem

  (1960, et.seq., P.D., et.al. '08, for HW)

$$\Rightarrow \boxed{\partial_t \{ (\overbrace{\text{WAD}}^{\text{Pseudomomentum}}) + \langle V_\theta \rangle \} = - \underbrace{\langle \tilde{V}_r \tilde{n} \rangle}_{\text{driving flux}} - \overbrace{\delta_t \langle \tilde{q}^2 \rangle / \langle q \rangle'}^{\text{local P.E. decrement}} - \underbrace{\nu \langle V_\theta \rangle}_{\text{drag}}}$$

WAD = Wave Activity Density, $\langle \tilde{q}^2 \rangle / \langle q \rangle'$

- pseudomomentum in θ-direction (Andrews, McIntyre '78)
- Generalized Wave Momentum Density

$\left\{ \begin{array}{l} \text{i) momentum of quasi-particle gas of waves, turbulence} \\ \text{ii) consequence of azimuthal/poloidal symmetry} \\ \text{iii) not restricted to linear response, but reduces correctly} \end{array} \right.$

- What Does it Mean ? $\rightarrow$ "Non-Acceleration Theorem":

$$\partial_t\{(\mathrm{WAD}) + \langle V_\theta\rangle\} = -\langle \tilde{V}_r \tilde{n}\rangle - \delta_t\langle \tilde{q}^2\rangle / \langle q\rangle' - \nu\langle V_\theta\rangle$$

  - absent $\begin{cases} \langle \tilde{V}_r \tilde{n}\rangle, & \text{driving flux} \\ \delta_t\langle \tilde{q}^2\rangle, & \text{local potential enstrophy decrement} \end{cases}$

    $\rightarrow$cannot $\begin{cases} \text{accelerate} \\ \text{maintain} \end{cases}$ Z.F. with stationary fluctuations!

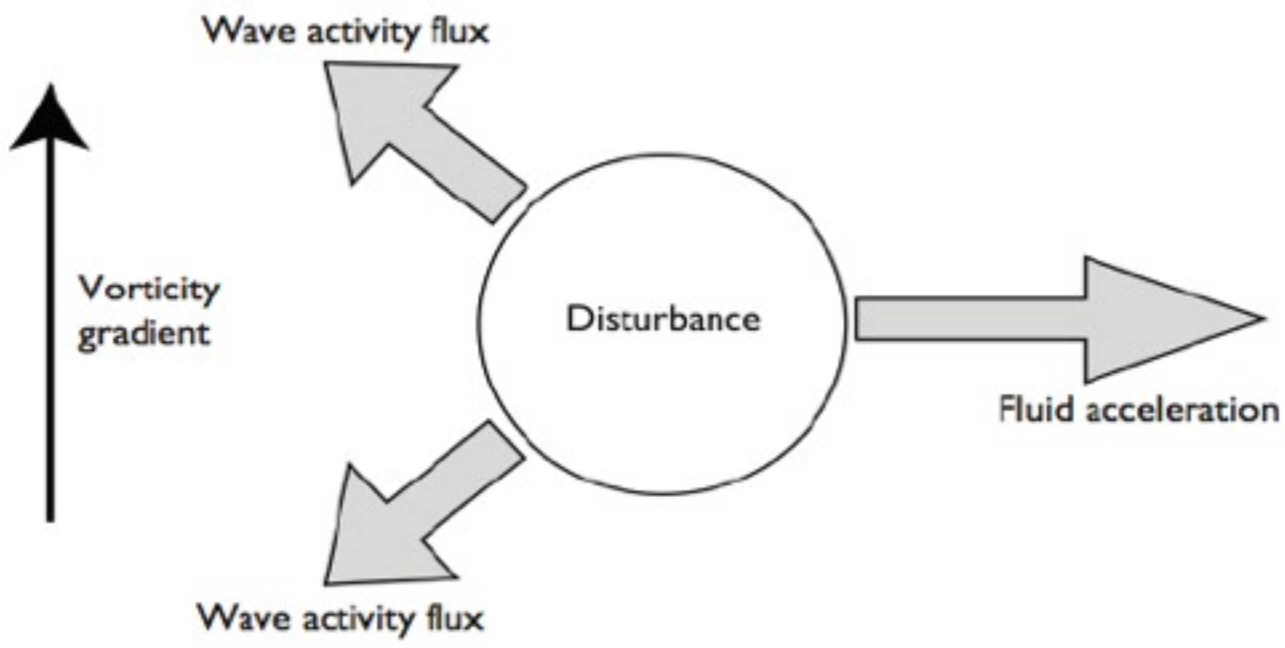
  - Essential physics is PV conservation and translational invariance in θ $\rightarrow$ freezing quasi-particle gas momentum into flow $\rightarrow$ relative "slippage" required for zonal flow growth

- obvious constraint on models of stationary zonal flows!

  $\leftrightarrow$ need explicit connection to relaxation, dissipation

  N.B. Inhomogeneous dissipation → incomplete homogenization!?

Aside: H-M



- C-D Theorem for HM

$$\partial_t\{\mathrm{WAD}+\langle V_\theta\rangle\}=\frac{\langle\tilde{f}^2\rangle\tau_c}{\langle q\rangle'}-\frac{1}{\langle q\rangle'}\left\{\partial_r\langle\tilde{V}_r\delta q^2\rangle+\mu\langle(\nabla\delta q)^2\rangle\right\}-\nu\langle V_\theta\rangle$$

- C-D prediction for $\langle V_\theta\rangle$ at stationary state, HM model

$$\langle V_\theta\rangle=\frac{1}{\nu\langle q\rangle'}\left\{\langle\tilde{f}^2\rangle\tau_c-\partial_r\langle\tilde{V}_r\delta q^2\rangle+\mu\langle(\nabla\delta q)^2\rangle\right\}$$

→ Note: Flow direction set by: $\langle q\rangle'$, source, sink distribution

→ Forcing, damping profiles determine shear

→ Potential Enstrophy Transport impact flow structure

In More Depth: What Really Determines Zonal Flow?

- driving flux: $\langle \tilde{V}_r \tilde{n} \rangle = \Gamma_0 - \Gamma_{\text{col}} = \int dr' S_n(r') - \Gamma_{\text{col}}$
  - Total flux Γ_0 fixed by sources, $S_n \rightarrow$ flux driven system
  - Collisional flux in turbulent system, Γ_{col} (computed with actual profiles)
  - $\Gamma_o - \Gamma_{col} \rightarrow$ available flux
  - P.E. decrement: $\delta_t \langle \tilde{q}^2 \rangle = \partial_r \langle \tilde{V}_r \tilde{q}^2 \rangle + D_0 \langle (\nabla \tilde{q})^2 \rangle$
    $\rightarrow$ change in roton intensity (PE) changes flow profile
    - roton dissipation
    - P.E. flux, direction increment, according to convergence (> 0) or divergence (< 0) of pseudomomentum, locally

    So: P.E. transport and "spreading" intrinsically linked to flow structure, dynamics

    Net δ(P.E.) can generate net spin-up

    $\therefore$ Zonal flow dynamics intrinsically "non-local" $\leftrightarrow$ couple to turbulence spreading (fast, meso-scale process)

Clarifying the Enigma of Collisionless Zonal Flow Saturation

- Flow evolution with: $\nu \to 0$, $S_n \neq 0$ and nearly stationary turbulence

$$\partial_t \langle V_\theta \rangle = -\left(\int dr' S_n(r') - \Gamma_{\mathrm{col}} \right) - \left(\partial_r \langle \tilde{V}_r \tilde{q}^2 \rangle + D_0 \langle (\nabla \tilde{q})^2 \rangle \right) / \langle q \rangle'$$

  Possible Outcomes:
  - $\langle q \rangle' \to 0$, locally $\to$ shear flow instability (the usual)
    $\leftrightarrow$ limit cycle of burst and recovery, effective viscosity?
    $\to$problematic with magnetic shear
  - $\langle \tilde{V}_r \tilde{n} \rangle$ v.s. $\partial_r \langle \tilde{V}_r \tilde{q}^2 \rangle$ $\to$ potential enstrophy transport and inhomogeneous turbulence, with $\tilde{n}/n \sim$ M.L.T
    $\to$ flux drive vs. roton population flux
    $\to$ novel saturation mechanism
  - $\langle q \rangle' \to 0$, globally $\to$ homogenized PV state (Rhines, Young, Prandtl, Batchelor)
    $\to$ decouples mean PV, PE evolution
  - homogeneous marginality, i.e. $\int dr' S_n(r') = \Gamma_{\mathrm{col}} \leftrightarrow$ ala' stiff core

N.B.: $\langle q \rangle' = 0 \Rightarrow \partial_r \langle n \rangle = \partial_r^2 \langle V_E \rangle = \partial_r \langle \omega_E \rangle$ → particular profile relation !

Summary of Flow Organization

concept: symmetry

process: PV mixing, transport

constraint released: Enstrophy conservation

players: drift waves

Mean Field: $\Gamma_{PV} = \langle \tilde{v}_r \tilde{q} \rangle$

Global Constraint: Bounding circulation

NL: Pseudomomentum Flux

Outcome: Zonal Flow Formation

Shortcoming: ZF pattern structure and collisionless saturation

Summary of comparison

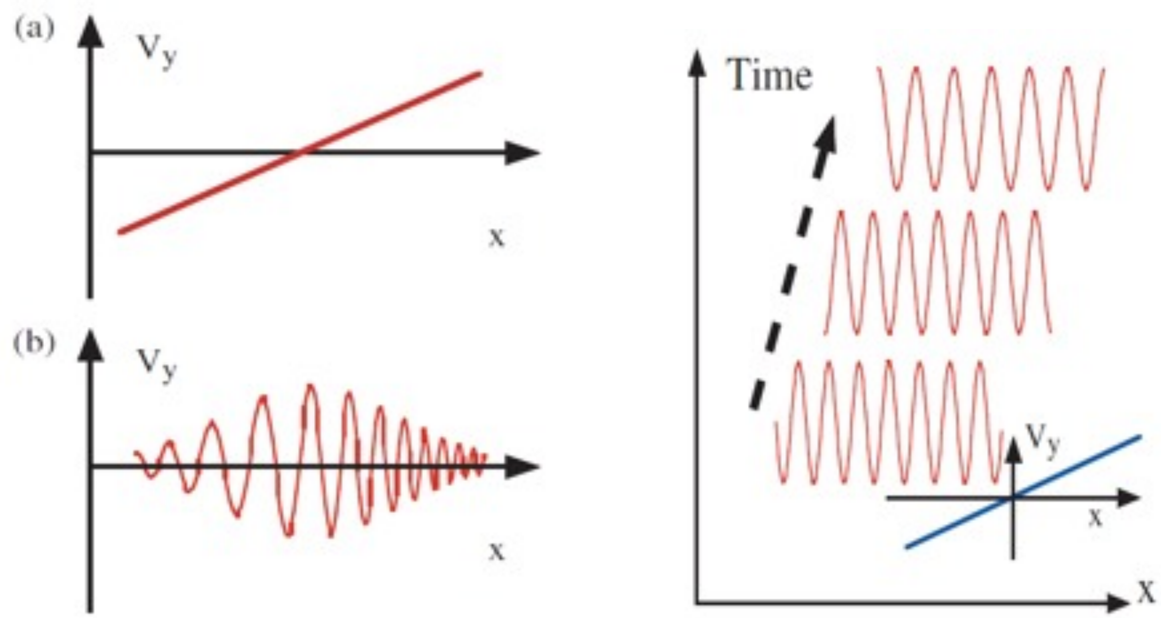
- Many commonalities between magnetic and flow relaxation apparent.

- Common weak point is limitation of mean field theory
  $\rightarrow$ difficult to grapple with strong NL, non-Gaussian fluctuations.

	Magnetic (JB)	Flow (GI)
concept	topology	symmetry
process	turbulent reconnection	PV mixing
players	tearing modes, Alfven waves	drift wave turbulence
mean field	EMF = $\langle \tilde{v} \times \tilde{B} \rangle$	PV Flux = $\langle \tilde{v}_r \tilde{q} \rangle$
constraint	$\int d^3x \mathbf{A} \cdot \mathbf{B}$ conservation	Potential Enstrophy balance
NL	Helicity Density Flux	Pseudomomentum Flux
outcome	B-profiles	zonal flow

Heuristics of Zonal Flows c.)

- One More Way:
- Consider:
  - –Radially propagating wave packet
  - –Adiabatic shearing field
- $$\frac{d}{dt}k_r = -\frac{\partial}{\partial r}\left(\omega + k_\theta \langle V_{E,ZF} \rangle\right) \Rightarrow \langle k_r^2 \rangle \uparrow$$
- $$\omega_{\vec{k}} = \frac{\omega_*}{1 + k_\perp^2 \rho_s^2} \downarrow$$



- Wave action density $N_k = E(k)/\omega_k$ adiabatic invariant
- $\therefore$ $E(k)\downarrow \Rightarrow$ flow energy decreases, due Reynolds work $\Rightarrow$ flows amplified (cf. energy conservation)
- $\Rightarrow$ Further evidence for universality of zonal flow formation

Heuristics of Zonal Flows d.) cont'd

- Implications:
  - –ZF's generic to drift wave turbulence in any configuration: electrons tied to flux surfaces, ions not
    - g.c. flux → polarization flux
    - zonal flow
  - –Critical parameters
    - ZF screening (Rosenbluth, Hinton '98)
    - polarization length
    - cross phase → PV mixing
- Observe:
  - –can enhance $e\varphi_{ZF}/T$ at fixed Reynolds drive by reducing shielding, ρ^2
  - –typically: $\boxed{\varepsilon/\varepsilon_0} \sim 1 + \rho_i^2/\lambda_D^2 + f_t \boxed{\rho_b^2}/\lambda_D^2 + f_d \boxed{\delta_d^2}/\lambda_D^2$

    total screening response

    banana width

    banana tip excursion
  - –Leverage (Watanabe, Sugama) → flexibility of stellerator configuration
    - Multiple populations of trapped particles
    - $\langle E_r \rangle$ dependence (FEC 2010)

Heuristics of Zonal Flows d.) cont'd

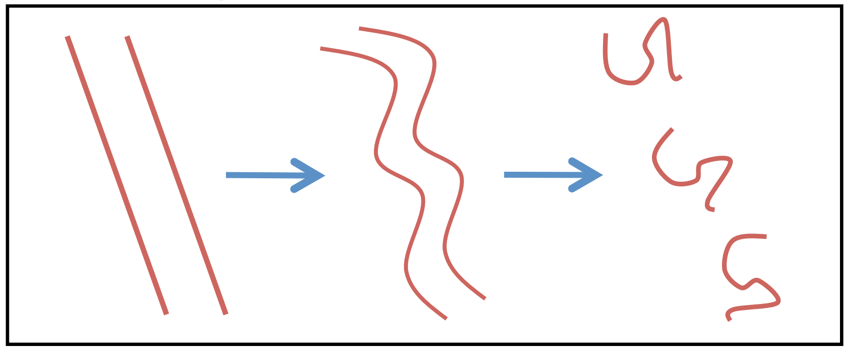
- Yet more: $\frac{\partial}{\partial t}\langle v_\perp\rangle = -\partial_r\langle \tilde{v}_{rE}\tilde{v}_{\perp E}\rangle - \boxed{\gamma_d\langle v_\perp\rangle} + \mu\nabla_r^2\langle v_\perp\rangle$

  damping

- Reynolds force opposed by flow damping
- Damping:
  - Tokamak → $\gamma_d \sim \gamma_{ii}$
    - trapped, untrapped friction
    - no Landau damping of (0, 0)
  - Stellerator/3D → $\gamma_d \leftrightarrow NTV$
    - damping tied to non-ambipolarity, also
    - largely unexplored
  - RMP
    - zonal density, potential coupled by RMP field
    - novel damping and structure of feedback loop
- Weak collisionality → nonlinear damping – problematic
  → tertiary → 'KH' of zonal flow →

  magnetic shear!?

  → other mechanisms?



Heuristics of Zonal Flows c.) cont'd

4) GAMs Happen

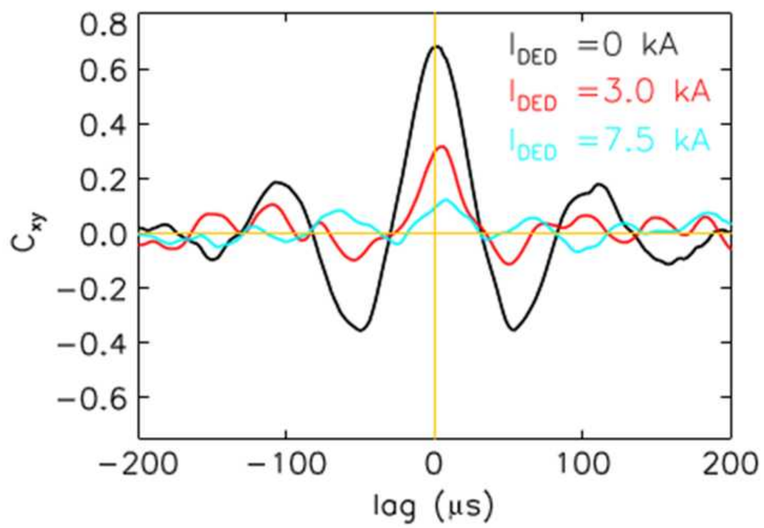
- Zonal flows come in 2 flavors/frequencies:
  - $\omega = 0 \Rightarrow$ flow shear layer
  - GAM $\omega^2 \cong 2c_s^2 / R^2 (1 + k_r^2 \rho_\theta^2) \Rightarrow$ frequency drops toward edge $\Rightarrow$ stronger shear
    - radial acoustic oscillation
    - couples flow shear layer (0,0) to (1,0) pressure perturbation
    - R ≡ geodesic curvature (configuration)
    - Propagates radially
- GAMs damped by Landau resonance and collisions

  $\gamma_d \sim \exp[-\omega_{GAM}^2 / (v_{thi} / Rq)^2]$
  - q dependence!
  - edge
- Caveat Emptor: GAMs easier to detect $\Rightarrow$ looking under lamp post ?!

Progress I: ZF's with RMP (with M. Leconte)

- ITER 'crisis du jour': ELM Mitigation and Control
- Popular approach: RMP
- ? Impact on Confinement?

Y. Xu '11



⇒ RMP causes drop in fluctuation LRC, suggesting reduced Z.F. shearing

⇒ What is "cost-benefit ratio" of RMP?

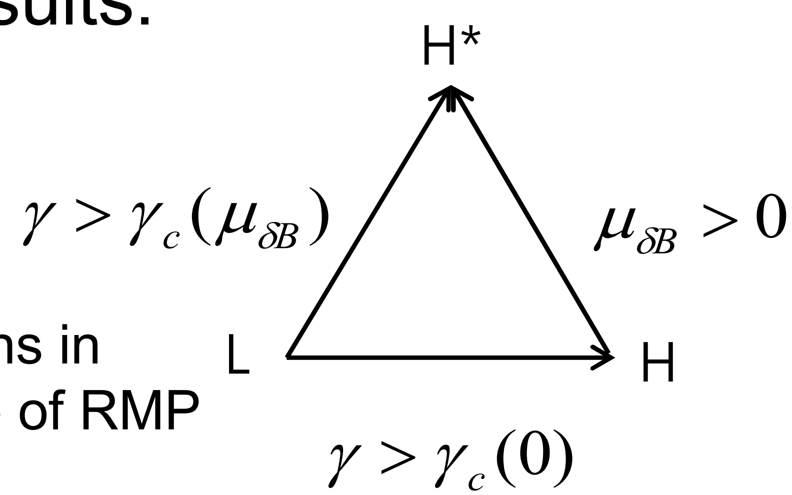
- Physics:
  - in simple H-W model, polarization charge in zonal annulus evolves according:

$$\frac{dQ}{dt} = -\int dA \left[\left\langle \tilde{v}_x \tilde{\rho}_{pol} \right\rangle + \left(\frac{\delta B_r}{B_0} \right)^2 D_{\parallel} \frac{\partial}{\partial x} \left(\langle \phi \rangle - \langle n \rangle \right) \right] \Bigg|_{r_1}^{r_2}$$

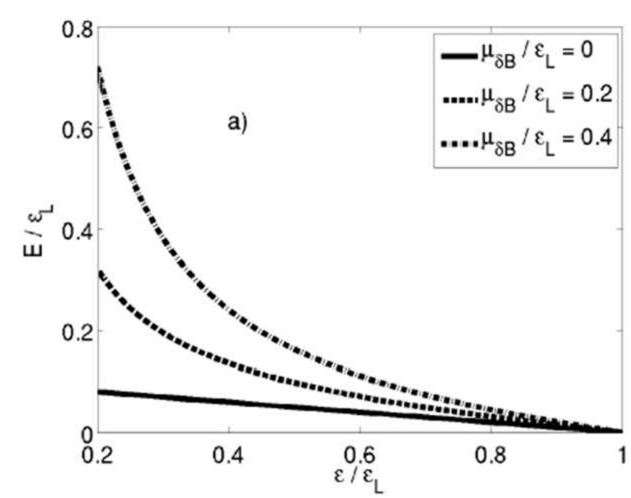
  - Key point: δB_r of RMP induces radial electron current → enters charge balance

Progress I, cont'd

- Implications
  - δB_r linearly couples zonal $\hat{\phi}$ and zonal $\hat{n}$
  - Weak RMP $\rightarrow$ correction, strong RMP $\rightarrow$ $\langle E_r \rangle_{ZF} \cong -T_e \partial_r \langle n \rangle / |e|$
- Equations: $$\frac{d}{dt}\delta n_q + D_T q^2 \delta n_q + i b_q (\delta\phi_q - (1-c)\delta n_q) - D_{RMP} q^2 (\delta\phi_q - \delta n_q) = 0$$
$$\frac{d}{dt}\delta\phi_q + \mu\delta\phi_q - a_q(\delta\phi_q - (1-c)\delta n_q) + \frac{D_{RMP}}{\rho_s^2}(\delta\phi_q - \delta n_q) = 0$$
- Results:



Transitions in presence of RMP



$E_{ZF}/\mathcal{E}_L$ vs $\mathcal{E}/\mathcal{E}_L$ for various RMP coupling strengths

Progress II : β-plane MHD (with S.M. Tobias, D.W. Hughes)

Model

- Thin layer of shallow magneto fluid, i.e. solar tachocline
- β-plane MHD ~ 2D MHD + β-offset i.e. solar tachocline

$$\partial_t \nabla^2 \phi + \nabla \phi \times \hat{z} \cdot \nabla \nabla^2 \phi - \nu \nabla^2 \nabla^2 \phi = \beta \partial_x \phi + B_0 \partial_x \nabla^2 A + \nabla A \times \hat{z} \cdot \nabla \nabla^2 A + \tilde{f}$$

$$\partial_t A + \nabla \phi \times \hat{z} \cdot \nabla A = B_0 \partial_x \phi + \eta \nabla^2 A \qquad \vec{B}_0 = B_0 \hat{x}$$

- Linear waves: Rossby – Alfven $\omega^2 + \omega\beta \frac{k_x}{k^2} - k_x^2 V_A^2 = 0$ (R. Hide)
- cf P.D., et al; Tachocline volume, CUP (2007)

  S. Tobias, et al: ApJ (2007)

Progress II, cont'd

Observation re: What happens?

- Turbulence → stretch field → $\langle \tilde{B}^2 \rangle >> B_0^2$ i.e. $\langle \tilde{B}^2 \rangle / B_0^2 \sim R_m$
  (ala Zeldovich)
- Cascades : - forward or inverse?
  - MHD or Rossby dynamics dominant !?
- PV transport: $\frac{dQ}{dt} = -\int dA \langle \tilde{v} \tilde{q} \rangle$ ⟶ net change in charge content due PV/polarization charge flux

Now $\frac{dQ}{dt} = -\int dA \left[\langle \tilde{v} \tilde{q} \rangle - \langle \tilde{B}_r \tilde{J}_{\parallel} \rangle \right] = -\int dA \partial_x \left\{ \langle \tilde{v}_x \tilde{v}_y \rangle - \langle \tilde{B}_x \tilde{B}_y \rangle \right\}$ ⟶ Reynolds mis-match

PV flux ↑ current along tilted lines ↑

⟶ vanishes for Alfvenized state

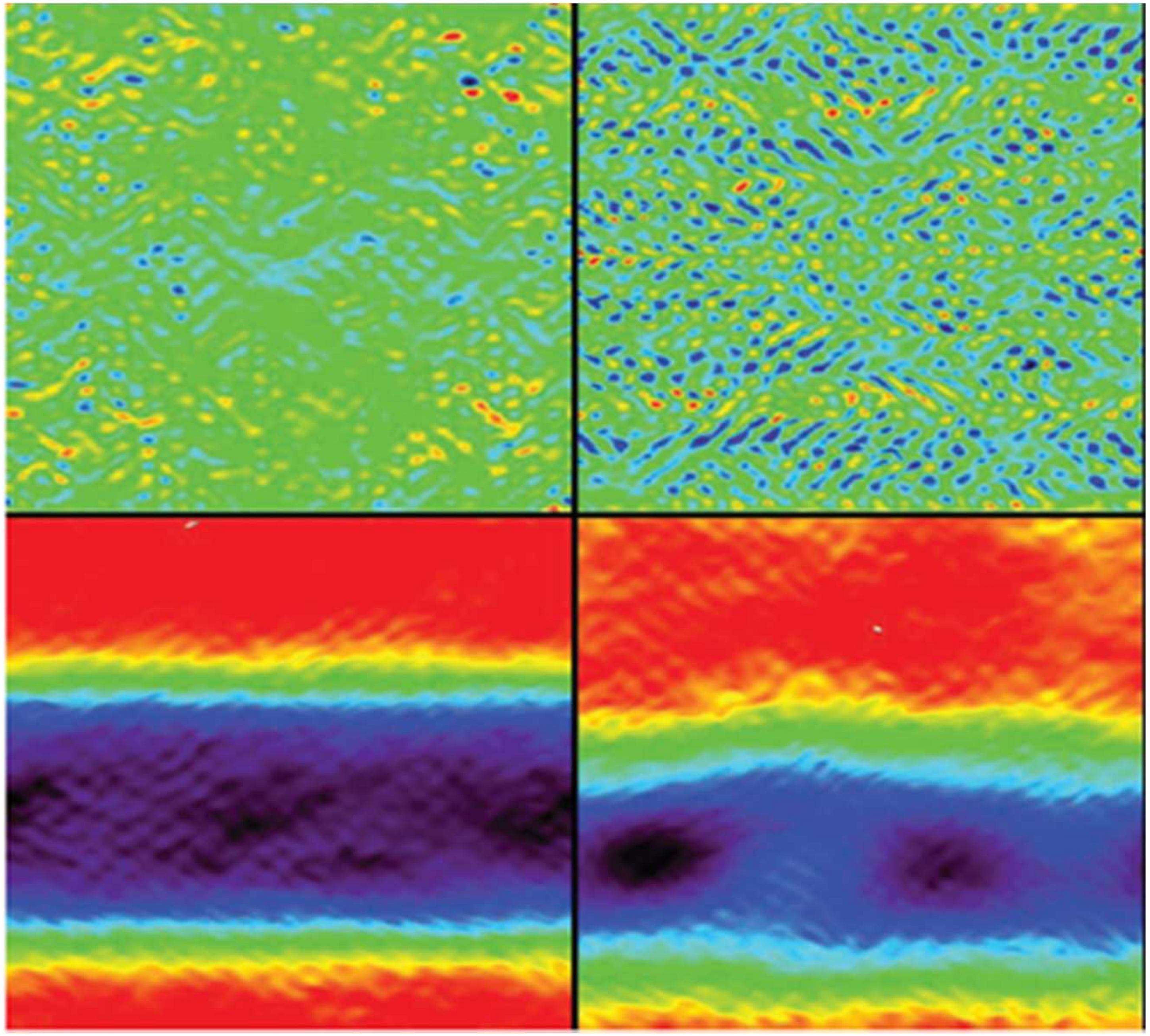
Taylor: $\langle \tilde{B}_x \tilde{J}_{\parallel} \rangle = -\partial_x \langle \tilde{B}_x \tilde{B}_y \rangle$

Progress II, cont'd

- **With Field**

$B_0 = 10^{-1}$

$B_0 = 10^{-2}$



$B_0 = 0$

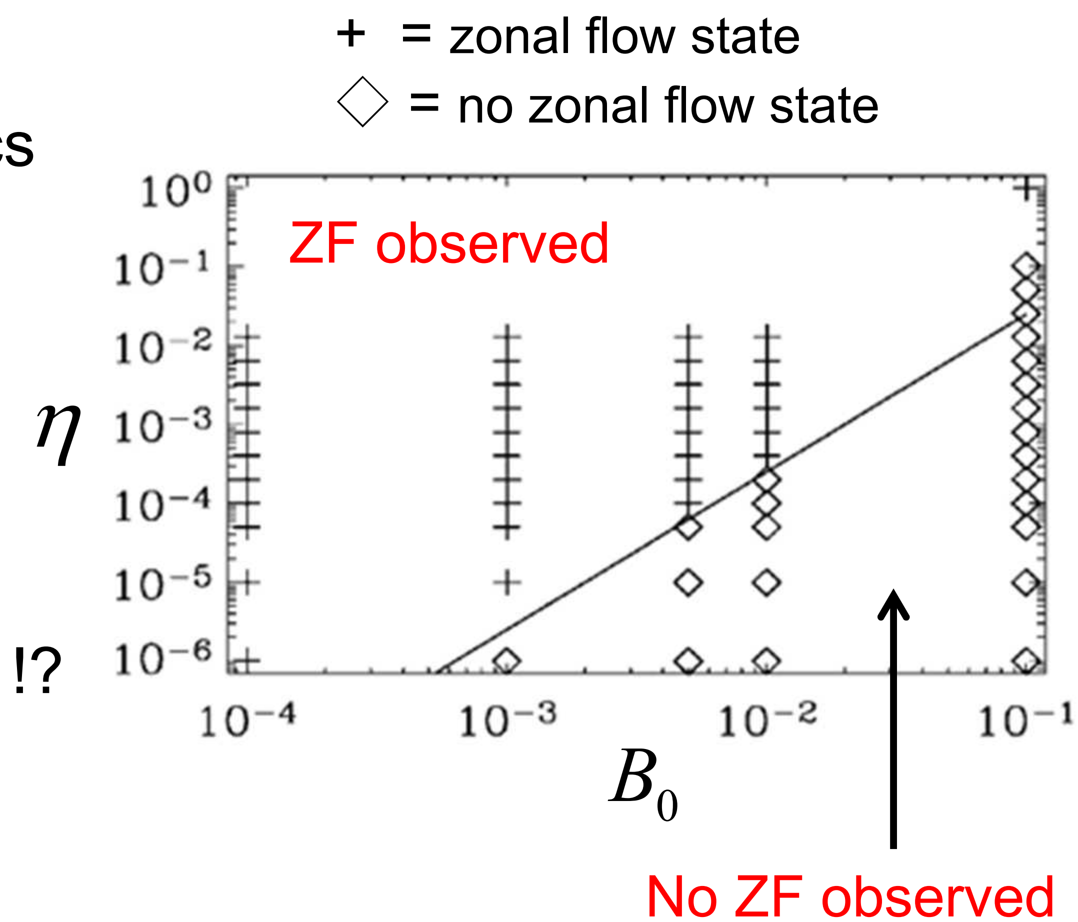
$B_0 = 10^{-3}$

Progress II, cont'd

- Control Parameters for $\tilde{\vec{B}}$ enter Z.F. dynamics

  Like RMP, Ohm's law regulates Z.F.

- Recall
  - $\langle \tilde{v}^2 \rangle$ vs $\langle \tilde{B}^2 \rangle$
  - $\langle \tilde{B}^2 \rangle \sim B_0^2 R_m$ $\longrightarrow$ origin of B_0^2 / η scaling !?



- Further study $\rightarrow$ differentiate between :
  - cross phase in $\langle \tilde{v}_r \tilde{q} \rangle$ and O.R. vs J.C.M
  - orientation : $\vec{B} \parallel \vec{V}$ vs $\vec{B} \perp \vec{V}$
  - spectral evolution