8-3. $v_{rms} = \sqrt{\frac{3RT}{M}}$

(a) For O_2: $v_{rms} = \sqrt{\frac{3(8.31J/K\bullet mol)(273K)}{32\times10^{-3}kg/mol}} = 461m/s$

(b) For H_2: $v_{rms} = \sqrt{\frac{3(8.31J/K\bullet mol)(273K)}{2\times10^{-3}kg/mol}} = 1840m/s$

8-6. $\langle v^2 \rangle = \frac{1}{N}\int_0^\infty v^2 n \ \ v \ \ dv = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\int_0^\infty v^4 e^{-\lambda v^2} dv$ where $\lambda = m/2kT$

$\langle v^2 \rangle = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2} I_4$ where I_4 is given in Table B1-1.

$$I_4 = \frac{3}{8}\pi^{1/2}\lambda^{-5/2} = \frac{3}{8}\pi^{1/2} \ \ m/2kT^{\ \ -5/2}$$

$$\langle v^2 \rangle = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\left(\frac{3}{8}\right)\pi^{1/2}\left(\frac{2kT}{m}\right)^{5/2} = \frac{3kT}{m} = \frac{3RT}{mN_A} = \frac{3RT}{M}$$

$$v_{rms} = \sqrt{\langle v^2 \rangle} = \sqrt{\frac{3RT}{M}}$$

8-9. $n\ v\ dv = 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-mv^2/2kT} dv$ (Equation 8-8)

$\frac{dn}{dv} = A\left[v^2\left(-\frac{2vm}{2kT}\right) + 2v\right]e^{-mv^2/2kT}$ The v for which $dn/dv = 0$ is v_m.

$$A\left[-\frac{2mv^3}{2kT} + 2v\right]e^{-mv^2/2kT} = 0$$

Because A = constant and the exponential term is only zero for $v \to \infty$, only the quantity in [] can be zero, so $-\frac{2mv^3}{2kT} + 2v = 0$

or $v^2 = \frac{2kT}{m} \to v_m = \sqrt{\frac{2kT}{m}}$ (Equation 8-9)

8-42. (a) $f(u)\,du = Ce^{-E/kT}du = Ce^{-Au^2/kT}du$ (from Equation 8-5)

$$1 = \int_{-\infty}^{+\infty} f(u)\,du = \int_{-\infty}^{+\infty} Ce^{-Au^2/kT}du = 2C\int_{-\infty}^{+\infty} e^{-Au^2/kT}du$$

$$= 2CI_0 = 2C\sqrt{\pi}\,\lambda^{-1/2}/2 \quad \text{where } \lambda = A/kT$$

$$= C\sqrt{\pi}\sqrt{kT/A} \rightarrow C = \sqrt{A/\pi kT}$$

(b) $$\langle E\rangle = \langle Au^2\rangle = \int_{-\infty}^{+\infty} Au^2 f(u)\,du = \int_{-\infty}^{+\infty} Au^2\sqrt{A/\pi kT}\,e^{-Au^2/kT}du$$

$$= A\sqrt{A/\pi kT}\,(2I_2) = A\sqrt{A/\pi kT}\,2\times(\sqrt{\pi}/4)\,\lambda^{-3/2} \quad \text{where } \lambda = A/kT$$

$$= \frac{1}{2}A\sqrt{A/kT}\,(kT/A)^{3/2} = \frac{1}{2}kT$$

3-12. $\lambda_m T = 2.898\times10^{-3}\, m\cdot K$

(a) $\lambda_m = \dfrac{2.898\times10^{-3}\, m\cdot K}{3K} = 9.66\times10^{-4}\, m = 0.966mm$

(b) $\lambda_m = \dfrac{2.898\times10^{-3}\, m\cdot K}{300K} = 9.66\times10^{-6}\, m = 9.66\mu m$

(c) $\lambda_m = \dfrac{2.898\times10^{-3}\, m\cdot K}{3000K} = 9.66\times10^{-7}\, m = 966nm$

3-13. Equation 3-4: $R = \sigma T^4$. Equation 3-6: $R = \dfrac{1}{4}cU$.

From Example 3-4: $U = \left(8\pi^5 k^4 T^4\right)/\left(15h^3c^2\right)$

$$\sigma = \frac{R}{T^4} = \frac{(1/4)cU}{T^4} = \frac{1}{4}c\left(8\pi^5 k^4 T^4\right)/\left(15h^3c^2T^4\right)$$

$$= \frac{2\pi^5\left(1.38\times10^{-23}\, J/K\right)^4}{15\left(6.63\times10^{-34}\, J\cdot s\right)^3\left(3.00\times10^{8}\, m/s\right)^2} = 5.67\times10^{-8}\, W/m^2K^4$$

3-14. Equation 3-18: $u(\lambda)=\dfrac{8\pi hc\lambda^{-5}}{e^{hc/\lambda kT}-1}$

$$u(f)df=u(\lambda)d\lambda \quad \therefore \quad u(f)=u(f)\frac{d\lambda}{df} \text{ Because } c=f\lambda, \quad \left|\frac{d\lambda}{df}\right|=c/f^2$$

$$u(f)=\frac{8\pi hc(f/c)^5}{e^{hf/kT}-1}\left(\frac{c}{f^2}\right)=\frac{8\pi f^2}{c^3}\frac{hf}{e^{hf/kT}-1}$$

3-15.

(a) $\lambda_m T=2.898\times10^{-3}m\cdot K \quad \therefore \quad \lambda_m=\dfrac{2.898\times10^{-3}m\cdot K}{2.7K}=1.07\times10^{-3}m=1.07mm$

(b) $c=f\lambda \quad \therefore \quad f=\dfrac{c}{\lambda_m}=\dfrac{3.00\times10^8 m/s}{1.07\times10^{-3}m}=2.80\times10^{11}Hz$

(c) Equation 3-6:

$$R=\frac{1}{4}cU=\frac{c}{4}\left(8\pi^5k^4T^4/15h^3c^3\right)$$
$$=\frac{(3.00\times10^8 m/s)(8\pi^5)(1.38\times10^{-23}J/K)^4(2.7)^4}{(4)(15)(6.63\times10^{-34}J\cdot s)^3(3.00\times10^8 m/s)^3}=3.01\times10^{-6}\ W/m^2$$

Area of Earth: $A=4\pi r_E^2=4\pi\left(6.38\times10^6m\right)^2$

Total power = $RA=\left(3.01\times10^{-6}W/m^2\right)(4\pi)\left(6.38\times10^6m\right)^2=1.54\times10^9W$

3-16. $\lambda_m T=2.898\times10^{-3}m\cdot K$

(a) $T=\dfrac{2.898\times10^{-3}m\cdot K}{700\times10^{-9}m}=4140K$

(b) $T=\dfrac{2.898\times10^{-3}m\cdot K}{3\times10^{-2}m}=9.66\times10^{-2}K$

(c) $T=\dfrac{2.898\times10^{-3}m\cdot K}{3m}=9.66\times10^{-4}K$

3-17. Equation 3-4: $R_1 = \sigma T_1^4$ $\quad R_2 = \sigma T_2^4 = \sigma\left(2T_1\right)^4 = 16\sigma T_1^4 = 16R_1$

3-18. (a) Equation 3-17: $\bar{E} = \dfrac{hc/\lambda}{e^{hc/\lambda kT}-1} = \dfrac{hc/\left(10hc/kT\right)}{e^{(hc/kT)/(10hc/kT)}-1} = \dfrac{0.1kT}{e^{0.1}-1} = 0.951kT$

(b) $\bar{E} = \dfrac{hc/\lambda}{e^{hc/\lambda kT}-1} = \dfrac{hc/\left(0.1hc/kT\right)}{e^{(hc/kT)/(0.1hc/kT)}-1} = \dfrac{10kT}{e^{10}-1} = 4.59\times10^{-4}kT$

Equipartition theorem predicts $\bar{E} = kT$. The long wavelength value is very close to *kT*, but the short wavelength value is much smaller than the classical prediction.

3-19. (a) $\lambda_m T = 2.898\times10^{-3}m\cdot K \quad \therefore \quad T_1 = \dfrac{2.898\times10^{-3}m\cdot K}{27.0\times10^{-6}m} = 107K$

$R_1 = \sigma T_1^4$ and $R_2 = \sigma T_2^4 = 2R_1 = 2\sigma T_1^4$

$\therefore \quad T_2^4 = 2T_1^4$ or $T_2 = 2^{1/4}T_1 = \left(2^{1/4}\right)\left(107K\right) = 128K$

(b) $\lambda_m = \dfrac{2.898\times10^{-3}m\cdot K}{128K} = 23\times10^{-6}m$

3-20. (a) $\lambda_m T = 2.898\times10^{-3}m\cdot K$ (Equation 3-5)

$$\lambda_m = \frac{2.898\times10^{-3}m\cdot K}{2\times10^4 K} = 1.45\times10^{-7}m = 145nm$$

(b) λ_m is in the ultraviolet region of the electromagnetic spectrum.

3-21. Equation 3-4: $R = \sigma T^4$

$P_{abs} = \left(1.36\times10^3 W/m^2\right)\left(\pi R_E^2 m^2\right)$ where R_E = radius of Earth

$P_{emit} = \left(RW/m^2\right)\left(4\pi R_E^2\right) = \left(1.36\times10^3 W/m^2\right)\left(\pi R_E^2 m^2\right)$

$$R = \left(1.36\times10^3 W/m^2\right)\left(\frac{\pi R_E^2}{4\pi R_E^2}\right) = \frac{1.36\times10^3}{4}\frac{W}{m^2} = \sigma T^4$$

$$T^4 = \frac{1.36\times10^3 W/m^2}{4\left(5.67\times10^{-8} W/m^2\cdot K^4\right)} \quad \therefore \quad T = 278.3K = 5.3°C$$

3-22. (a) $\lambda_m T = 2.898\times10^{-3} m\cdot K \quad \therefore \quad \lambda_m = \dfrac{2.898\times10^{-3} m\cdot K}{3300K} = 8.78\times10^{-7} m = 878nm$

$$f_m = c/\lambda_m = \frac{3.00\times10^8 m/s}{8.78\times10^{-7} m} = 3.42\times10^{14} Hz$$

(b) Each photon has average energy $E = hf$ and $NE = 40J/s$.

$$N = \frac{40J/s}{hf_m} = \frac{40J/s}{\left(6.63\times10^{-34} J\cdot s\right)\left(3.42\times10^{14} Hz\right)} = 1.77\times10^{20}\ photons/s$$

(c) At $5m$ from the lamp N photons are distributed uniformly over an area $A = 4\pi r^2 = 100\pi m^2$. The density of photons on that sphere is $\left(N/A\right)/s\cdot m^2$.

The area of the pupil of the eye is $\pi\left(2.5\times10^{-3} m\right)^2$, so the number of photons entering the eye per second is:

$$n = \left(N/A\right)\left(\pi\right)\left(2.5\times10^{-3} m\right)^2 = \frac{\left(1.77\times10^{20}/s\right)\left(\pi\right)\left(2.5\times10^{-3} m\right)^2}{100\pi m^2}$$
$$= \left(1.77\times10^{20}/s\right)\left(\pi\right)\left(2.5\times10^{-3} m\right)^2 = 1.10\times10^{13}\ photons/s$$