Physics 4E Quiz 4 Solutions Spring 2015

Problem 1

Assuming V_0 is large (check later), the energy of this electron is

$$E = \frac{\hbar^2 \pi^2}{2mL_1^2} = \frac{3.81\pi^2}{25} eV = 1.50 eV$$

The speed of this electron is:

$$v = \frac{p}{m} = \frac{\hbar k}{m} = \frac{\hbar \pi}{m L_1} \Rightarrow \frac{v}{c} = \frac{\hbar c \pi}{mc^2 L_1} = 0.0024$$

$$\Rightarrow \boxed{v = 0.0024c = 7.27 \times 10^{14} \frac{Å}{s}}$$

the probability that the electron will tunnel through the barrier when it hits it is

$$T = e^{-2S\Delta x}, \quad S = \sqrt{\frac{2m}{\hbar^2}(V_0 - E)}, \quad \Delta x = L_2 - L_1 = 3Å$$

The time it takes the electron to go back and forth once is

$$t = \frac{2L_1}{v}$$

$\Rightarrow$ the number of times per second it tries to tunnel is $N = \frac{1}{t} = \frac{v}{2L_1}$

The mean lifetime τ is given by $\frac{1}{\tau} = NT$

We have $\tau = 1s \Rightarrow T = \frac{t}{\tau} = \frac{2 \times 5}{7.27 \times 10^{14}} = 1.38 \times 10^{-14}$

$$\Rightarrow -2\sqrt{\frac{2m}{\hbar^2}(V_0 - E)}\,\Delta x = -15.96 \Rightarrow V_0 - E = 107.81 eV \Rightarrow$$

$\boxed{V_0 = 109.31 eV}$ since $V_0 >> E$, infinite well approximation is good

(b) For $\tau' = 10s \Rightarrow \frac{e^{2S \cdot \Delta x'}}{e^{2S\Delta x}} = 10 \Rightarrow 2S(L_2' - L_2) = \ln 10 \Rightarrow$

$L_2' = L_2 + \frac{\ln 10}{2S}$, $S = 5.32 Å^{-1} \Rightarrow \boxed{L_2' = 8.22 Å}$

Problem 2

(a) $$\psi(r,\theta,\phi) = C\, r^3 e^{-r/a_0} f(\theta)\, g(\phi)$$

Exponential part of radial wavefunction is always e^{-Zr/na_0} $\Rightarrow$

for this wavefunction $\boxed{n = Z}$

Energy is $E = -E_0 \frac{Z^2}{n^2} \Rightarrow \boxed{E = -E_0 = -13.6\,eV}$

(b) From the form of the radial wavefunction $\Rightarrow$ $n = 4$ (r^3 factor)

$\Rightarrow \boxed{Z = 4}$

(c) From the form of the radial wavefunction (no nodes) $\Rightarrow$ l has maximum value $\Rightarrow$ $l = 3$. The possible m values are $-3, -2, -1, 0, 1, 2, 3$

(d) Shortest wavelength photon it can emit: selection rule $\Delta l = \pm 1$ $\Rightarrow$ has to go to $n = 3$, $l = 2$ state $\Rightarrow$

$$\Delta E = \frac{hc}{\lambda} = E_0 Z^2\left(\frac{1}{9} - \frac{1}{16}\right) = E_0 \cdot \frac{7}{9} \Rightarrow \lambda = \frac{9}{7}\,\frac{12{,}400}{13.6}\,\text{Å}$$

$\Rightarrow \boxed{\lambda = 1172\,\text{Å}}$

Problem 3

$$P(r) = C r^2 R^2(r) = C r^8 e^{-2r/a_0}$$

Most probable r: $P'(r) = 0 \Rightarrow$

$$8r^7 - \frac{2r^8}{a_0} = 0 \Rightarrow \boxed{r = 4a_0 \equiv r_m}$$

Bohr orbit for $n=4, Z=4$:

$$r_n = n^2 \frac{a_0}{Z} = 4a_0 \Rightarrow \text{same as most probable } r$$

(b) $$\langle \tfrac{1}{r} \rangle = \frac{\int_0^\infty dr\, r^7 e^{-2r/a_0}}{\int_0^\infty dr\, r^8 e^{-2r/a_0}} = \frac{7!\, a_0^8\, 2^9}{2^8\, 8!\, a_0^9} = \frac{2}{8a_0} = \frac{1}{4a_0}$$

$$\Rightarrow \boxed{\langle \tfrac{1}{r} \rangle = \frac{1}{4a_0} = \frac{1}{r_m}}$$

(c) $$\langle r \rangle = \frac{\int_0^\infty dr\, r^9 e^{-2r/a_0}}{\int_0^\infty dr\, r^8 e^{-2r/a_0}} = \frac{9!\, a_0^{10}\, 2^9}{2^{10}\, 8!\, a_0^9} = \frac{9}{2} a_0$$

$$\Rightarrow \boxed{\langle r \rangle = \frac{9}{2} a_0} \quad \text{or} \quad \boxed{\langle r \rangle = 1.125\, r_m}$$