Use: $\hbar^2/2m_e = 3.81 eVA^2$, m_e=electron mass

Problem 1 (10 pts)
The vibrations of a one-dimensional solid are modeled by assuming that each atom is a harmonic oscillator of frequency $\omega = 0.1 eV/\hbar$.
(a) Find the average energy of one oscillator at temperatures (i) kT=1eV, (ii) kT=0.1eV, (iii) kT=0.01eV. Include the zero point energy.
(b) Find the average energy of one oscillator for temperature kT=1.1eV, and using this and the result found in (a) (i) for kT=1eV calculate (approximately by numerical differentiation) the heat capacity per mol in units of R for kT=1eV.
(c) Estimate without doing a calculation the heat capacity per mol in units of R for kT=10eV. Justify your answer.
(d) Calculate approximately the heat capacity per mol in units of R for kT=0.01eV.

Problem 2 (10 pts)
An electron in a hydrogen atom is in a state with quantum number n=3.
(a) Ignoring electron spin, how many different energies can this electron have in the presence of a magnetic field of 100T? (T=tesla) Give the difference in the energy of the lowest and highest state, in eV.
(b) Same as (a) including electron spin but ignoring the spin-orbit interaction.
(c) For a system of many hydrogen atoms with the electron in the n=3 state in a magnetic field of 100T at temperature T=300K, what is the ratio of the number of atoms in the highest energy state to the number of atoms in the lowest energy state? Include electron spin but ignore the spin-orbit interaction.

Problem 3 (10 pts)
Consider a system of N classical particles confined in a two-dimensional square area of sides L. Treat the square as an infinite square well.
(a) Following the steps in the book leading to Eq. 8-41, find the equivalent expression for this case, i.e. the density of states g(E) for a two-dimensional system of area $A=L^2$.
(b) Find an expression for e^α for this system, analogous to Eq. 8-44 for a three-dimensional system. Using this result, give the normalized Boltzmann distribution f_B, Eq. 8-26 for this system.
(c) Give an expression for $n(E)=g(E)f_B(E)$ for this system and show by direct calculation that it is properly normalized.

Justify all your answers to all problems.