The Nature of Scientific Progress, More Error Analysis, Exp #2

Lecture # 3

Physics 2BL

Summer 2015

Outline

- Last time introduced significant figures, standard deviations, standard deviation of the mean
- Today instigate clicker questions
- discuss how scientific knowledge progresses: replacing models, restricting models
- What you should know about error analysis (so far) and more
- Introduce limiting Gaussian distribution
- Exp. 1
- Reminder

Models

- Invented
- Properties correspond closely to real world
- Must be testable

How Models Fit Into Process of Doing Science

- Science is a process that studies the world by:
  - Limiting the focus to a specific topic (***making a choice***)
  - Observing (***making a measurement***)
  - Refining Intuitions (***making sense***) Creating
  - Extending (***seeking implications***) Predicting
  - Demanding consistency (***making it fit***) Refining or Replacing
  - Community evaluation and critique

- Start with simple model

How Models Change

- If models disagree with observation, we change the model
  - Refine - add to existing structure
  - Restrict - limit scope of utility
  - Replace - start over

Refining

- Original model consistent with observations, but not complete
- Extend model to account for new observations
- May include new concepts

  e.g. Model of interaction between charged objects; to include interactions between charged & uncharged add concept of induced charge

Restriction

- New model correct in situations where old isn't
- New model agrees w/ old over some range

⇨ Old still useful in limited range

e.g. General relativity vs. classical gravitational theory

Replacement

- Old model can’t be extended consistently
- Replace entire model

⇨ Earlier observations provide limits for new model

e.g. Geocentric vs heliocentric models for solar system

Random and independent?

Yes	**No**
• Estimating between marks on ruler or meter • Releasing object from ‘rest’ • Mechanical vibration o **Judgment** o **Problems of definition**	• End of ruler screwy • Reading meter from the side (speedometer effect) • Scale not zeroed Reaction time delay o **Calibration** o **Zero**

Random & independent errors:

$$q = x + y - z$$

$$\delta q = \sqrt{(\delta x)^2 + (\delta y)^2 + (\delta z)^2}$$

$$q = Bx$$

$$\delta q = |B| \delta x$$

$$\frac{\delta q}{|q|} = \frac{\delta x}{|x|}$$

$$q = x \times y \div z$$

$$\frac{\delta q}{|q|} = \sqrt{\left(\frac{\delta x}{x}\right)^2 + \left(\frac{\delta y}{y}\right)^2 + \left(\frac{\delta z}{z}\right)^2}$$

$$q = q(x, y, z)$$

$$\delta q = \sqrt{\left(\frac{\partial q}{\partial x}\delta x\right)^2 + \left(\frac{\partial q}{\partial y}\delta y\right)^2 + \left(\frac{\partial q}{\partial z}\delta z\right)^2}$$

Propagation in formulas

Independent

Propagate error in steps

For example:

$$q = \frac{x}{y - z}$$

- First find

$$p = y - z$$

$$\delta p = \sqrt{(\delta y)^2 + (\delta z)^2}$$

- Then

$$q = \frac{x}{p}$$

$$\frac{\delta q}{|q|} = \sqrt{\left(\frac{\delta x}{x}\right)^2 + \left(\frac{\delta p}{p}\right)^2}$$

An Important Simplifying Point

$$h = \frac{1}{2} g t^2$$

$$g = 2h/t^2, \; \delta h/h = 5\%, \; \delta t/t = 0.1\%$$

$$\frac{\delta g}{g} = \sqrt{\left(\frac{\delta h}{h}\right)^2 + \left(2\frac{\delta t}{t}\right)^2}$$

$$\delta g/g = \sqrt{5\%^2 + (2 \times 0.1\%)^2}$$

$$\delta g/g = 0.050039984 = 5\%$$

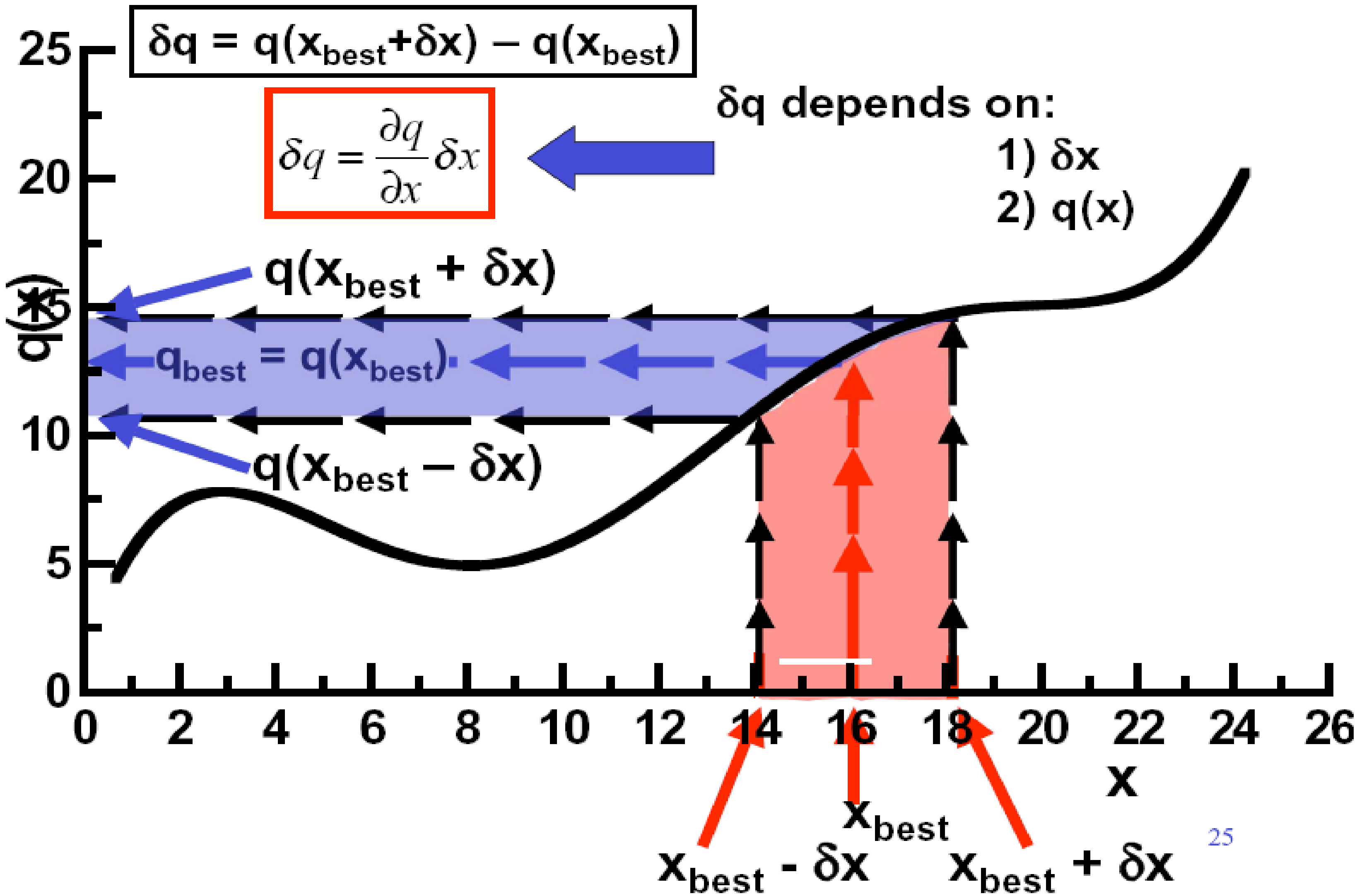
Requires random & ind. errors!

- Often the error is dominated by error in least accurate measurement

⇨Simplifies calc.

⇨Suggests improvements in experiment

Error Propagation



From Yagil

General Formula for error propagation

For independent, random errors

$$\delta q = \left| \frac{dq}{dx} \right| \delta x$$

$$\delta q = \sqrt{\left(\frac{\partial q}{\partial x} \delta x \right)^2 + \left(\frac{\partial q}{\partial y} \delta y \right)^2}$$

From Yagil

Analyzing Multiple Measurements

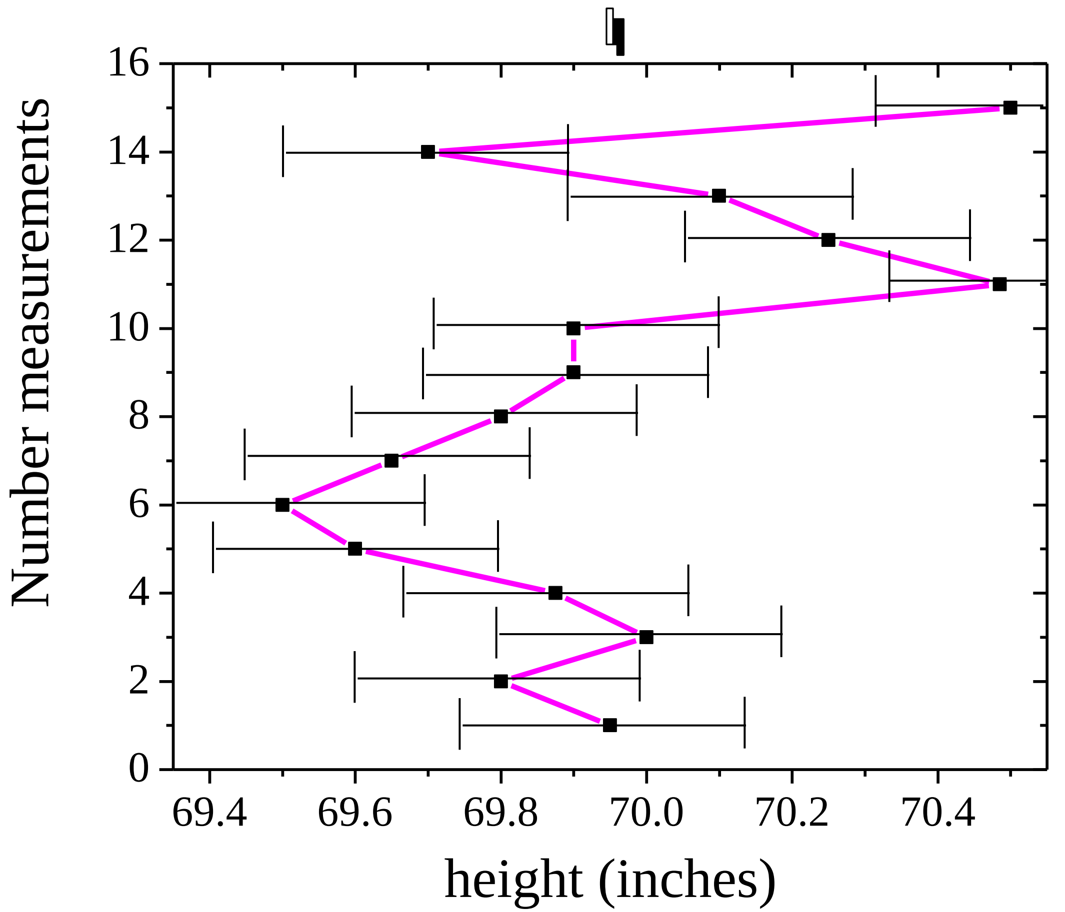
- Repeat measurement of x many times
- Best estimate of x is average (mean)

$$x_1, x_2, \cdots, x_N$$

$$x_{best} = \bar{x} = \frac{x_1 + x_2 + \cdots + x_N}{N}$$

$$\bar{x} = \frac{\sum x_i}{N}$$

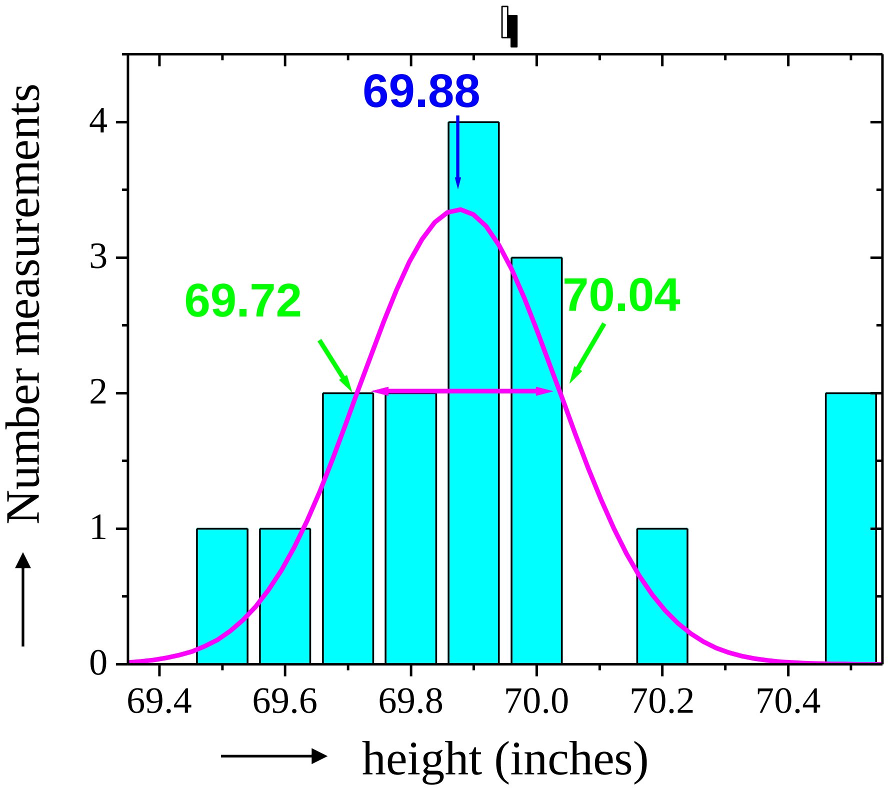
Repeated Measurements



How are Measured Values Distributed?

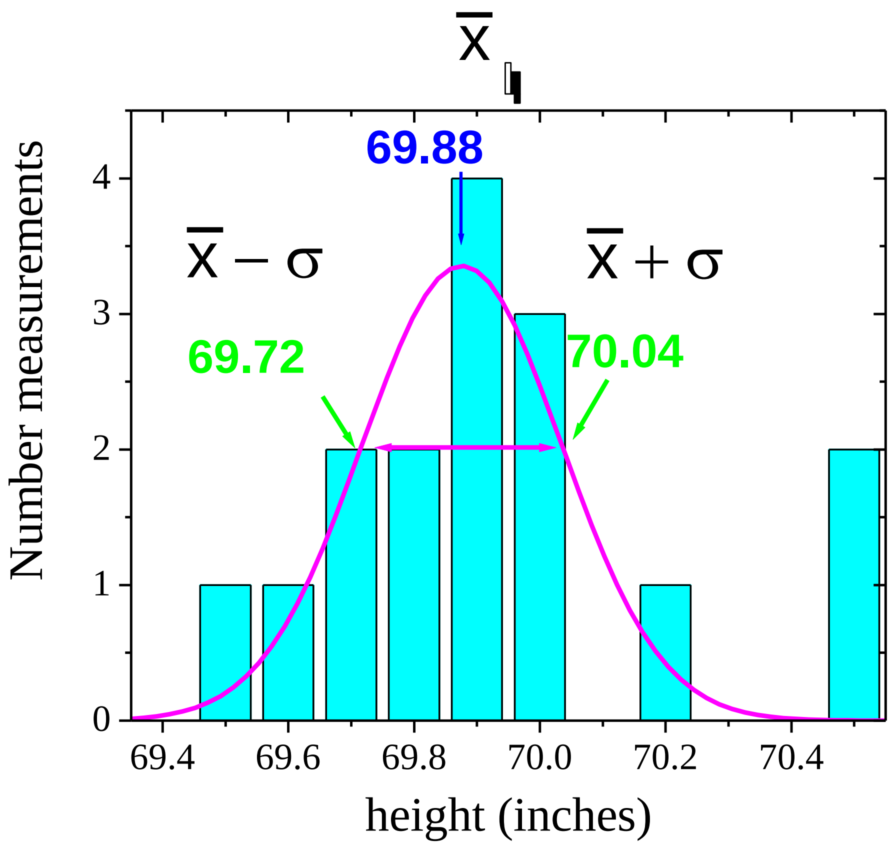
- If errors are random and independent:
  - Expect most values near true value
  - Expect few values far from true value

⇨ Assume values are distributed *normally*



$$G_{X,\sigma}(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-(x-\overline{x})^2 / 2\sigma^2\right)$$

Normal Distribution



$$G_{X,\sigma}(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-(x-\bar{x})^2 / 2\sigma^2\right)$$

Error of an Individual Measurement

- How precise are measurements of x?
- Start with each value's deviations from mean
- Deviations average to zero, so square, then average, then take square root
- ~68% of time, x_i will be w/in σ_x of true value

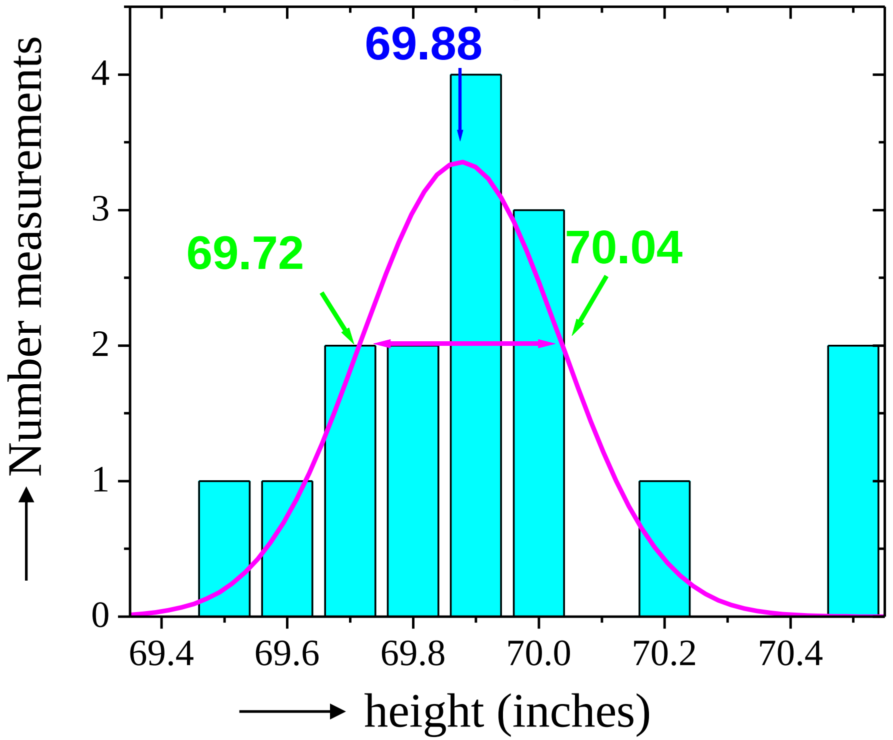
$$d_i \equiv x_i - \bar{x}$$

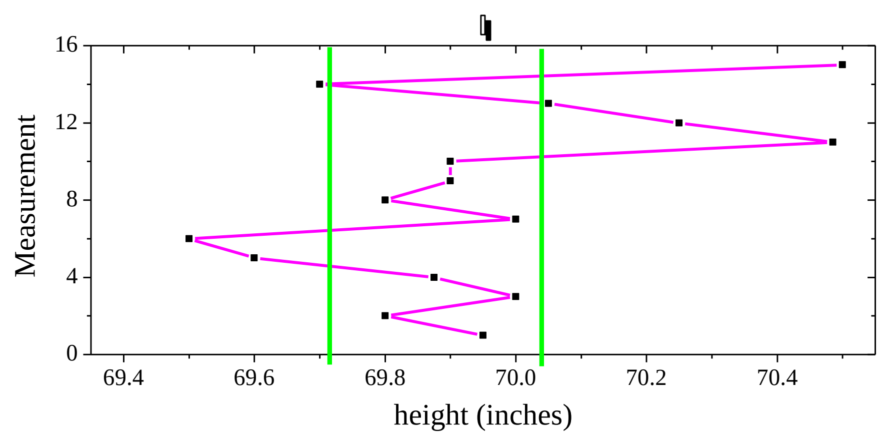
$$\bar{d} = 0$$

$$\sigma_x \equiv \sqrt{\overline{(d_i)^2}} = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

Take σ_x as error in individual measurement - called standard deviation

Standard Deviation





Drawing a Histogram

1. Determine the range of your data by subtracting the smallest number from the largest one.

2. The number of bins should be approximately $\sqrt{N}$ and the width of a bin should be the range divided by $\sqrt{N}$.

3. Make a list of the boundaries of each bin and determine which bin each of your data points should fall into.

4. Draw the histogram. The y axis should be the number of values that fall into each bin.

5. Sometimes this procedure will not produce a good histogram. If you make too many bins the histogram will be flat and too few bins will not show the curve on either side of the maximum. You might need to play around with the number of bins to produce a better histogram.

Error of the Mean

- Expect error of mean to be lower than error of the measurements it's calculated from
- Divide SD by square root of number of measurements
- Decreases slowly with more measurements

Standard Deviation of the Mean (SDOM)
or
Standard Error
or
Standard Error of the Mean

$$\sigma_{\bar{x}} = \sigma_x / \sqrt{N}$$

Summary

- Average

$$\bar{x} = \frac{\sum x_i}{N}$$

- Standard deviation

$$\sigma_x = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

- Standard deviation of the mean

$$\sigma_{\bar{x}} = \sigma_x / \sqrt{N}$$

The Four Experiments

- **Determine the average density of the earth**

Weigh the Earth, Measure its volume

– Measure simple things like lengths and times

– Learn to estimate and propagate errors

- **Non-Destructive measurements of densities, inner structure of objects**

– Absolute measurements *vs.* Measurements of variability

– Measure moments of inertia

– Use repeated measurements to reduce random errors

- **Construct and tune a shock absorber**

– Adjust performance of a mechanical system

– Demonstrate critical damping of your shock absorber

- **Measure coulomb force and calibrate a voltmeter.**

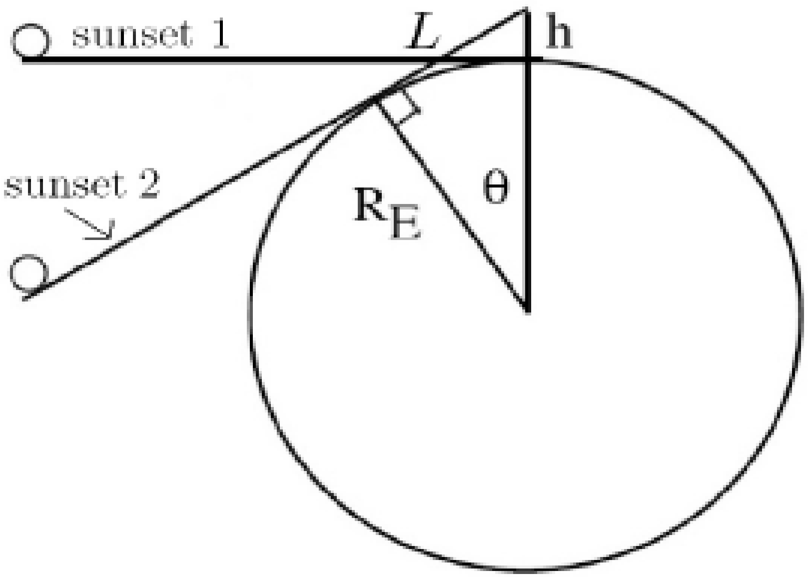
– Reduce systematic errors in a precise measurement.

The Earth

Volume – radius
Mass
Density

Experiment 1 Overview: Density of Earth

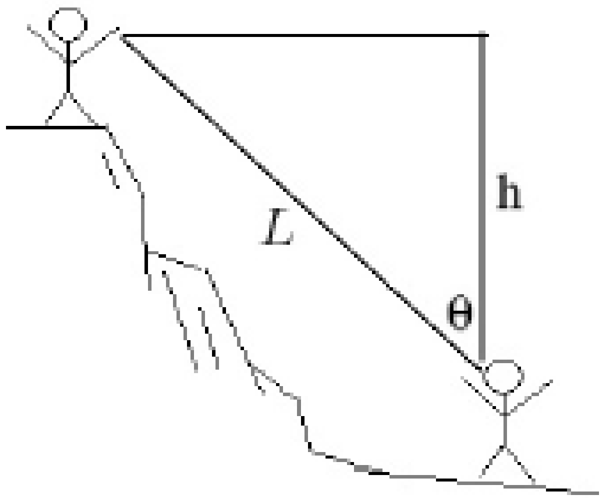
density $$\rho = \frac{M_E}{\frac{4}{3}\pi R_E^3} = \frac{3g}{4\pi G R_E}$$ $$= \frac{G M_E m}{R_E^2} = mg$$

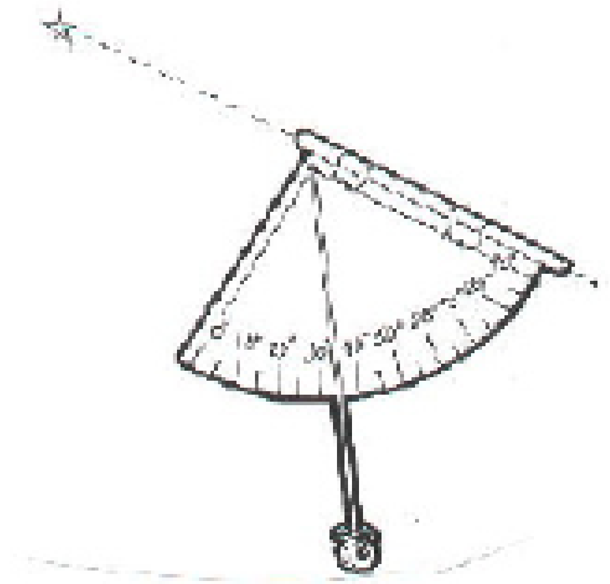


$$R_E = \frac{2h}{\omega^2 (\Delta t)^2}$$

measure Δt between sunset on cliff and at sea level

Experiment 1: Height of Cliff





rangefinder to get L

Sextant to get θ

Wear comfortable shoes

Make sure you use θ and not (90 – θ)

Measure Earth's Radius using Δt Sunset

Now, is this time delay measurable?

h - height above the sea level

L - distance to the horizon line

$$t = \frac{L}{2\pi R_e} T = \frac{T}{2\pi}\sqrt{\frac{2h}{R_e}}$$

$$T = 24\,\mathrm{hr} = 24 \cdot 60 \cdot 60\ \mathrm{s}$$

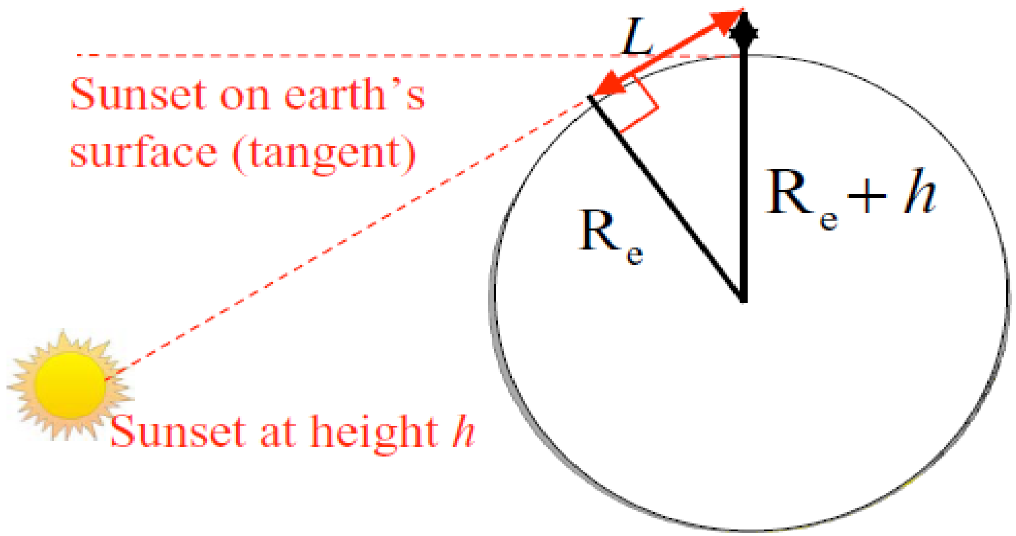
$$= 86400\ \mathrm{s}$$

$$R_e = 6{,}000{,}000\ \mathrm{m}$$

$h \sim 100$ m - our cliff

$$t = \frac{86400\ \mathrm{s}}{2\pi}\sqrt{\frac{200}{6 \times 10^6}} \approx 80\ \mathrm{s}$$

Looks doable!



Have we forgotten something?

From Yagil

"The Equation" for Experiment 1a

$$t = \frac{T}{2\pi}\sqrt{\frac{2Ch}{R_e}} = \frac{1}{\omega}\sqrt{\frac{2Ch}{R_e}}$$

from previous page.

$$\omega = \frac{2\pi}{24 \text{ hr}}$$

Which are the variables that contribute to the error significantly?

$$\Delta t = t_1 - t_2 = \frac{1}{\omega}\sqrt{\frac{2\overline{C}}{\mathrm{R_e}}}\left(\sqrt{h_1} - \sqrt{h_2}\right)$$

Time difference between the two sunset observers.

$$C \equiv \frac{1}{\cos^2(\lambda)\cos^2(\lambda_s) - \sin^2(\lambda)\sin^2(\lambda_s)}$$

Season dependant factor slightly greater than 1.

What other methods could we use to measure the radius of the earth?

The formula for your error analysis.

$$\mathrm{R_e} = \frac{2C}{\omega^2}\left(\frac{\sqrt{h_1} - \sqrt{h_2}}{\Delta t}\right)^2$$

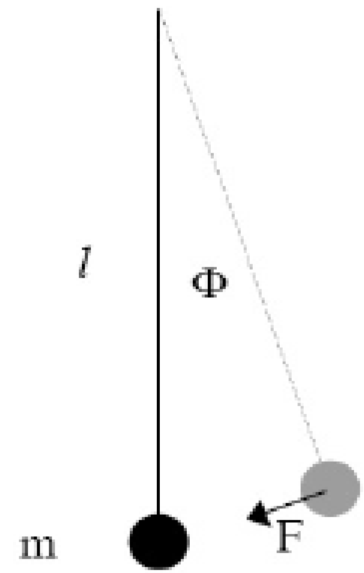
Eratosthenes

angular deviation = angle subtended

From Yagil

Experiment 1: Determine g

pendulum



$F = -mg\sin(\phi) = -mg\phi$

$F = m\alpha = ml\ddot{\phi}$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{g}}$$

period

Experiment 1: Pendulum

- For release angle θ_i, you should have a set of time data $(t_1^p, t_2^p, t_3^p, ..., t_N^p)$.

- Calculate the average, $\bar{t^p}$, and the the standard deviation, σ_{t^p}, of this data.

- Divide $\bar{t^p}$ and σ_{t^p} by p to get average time of a *single* period, $\bar{T}$ and standard deviation of a single period σ_T.

- Calculate SDOM, $\sigma_{\bar{T}} = \frac{\sigma_T}{\sqrt{N}}$.

- Now you should have $\bar{T} \pm \sigma_{\bar{T}}$ for you data at θ_i.

- Repeat these calculations for data at each release angle.

Grading rubric uploaded on website

Error Propagation - example

Ws saw earlier how to determine the acceleration of gravity, g.

Using a simple pendulum, measuring its length and period:

-Length l : $l = l_{best} \pm \delta l$

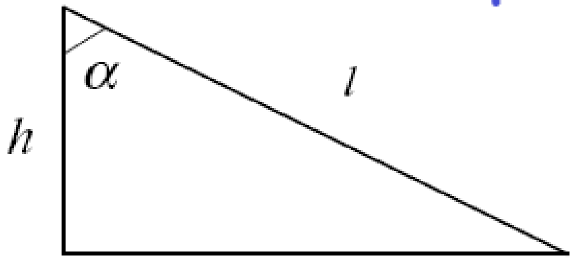
-Period T : $T = T_{best} \pm \delta T$

Determine g by solving: $g = l \cdot (2\pi / T)^2$

The question is what is the resulting uncertainty on g, **δg** ??

From Yagil

Example



Given that: $l = 10 \pm 0.1$ m

$\alpha = 20 \pm 3^0$

==> Find *h*.

$$h = l \cdot \cos\alpha = 10 \cdot \cos 20^0 = 10 \cdot 0.94 = 9.4 \text{ m}$$

$$\delta h = \sqrt{\left(\frac{\partial h}{\partial l}\delta l\right)^2 + \left(\frac{\partial h}{\partial \alpha}\delta\alpha\right)^2}$$

$$\frac{\partial h}{\partial l} = \cos\alpha$$

$$\frac{\partial h}{\partial \alpha} = l \cdot (-\sin\alpha)$$

always use radians when calculating the errors on trig functions

$$\delta\alpha = 3^0 = \frac{2\pi \text{ rad}}{360^0} \cdot 3^0 = 0.05 \text{ rad}$$

$$\delta h = \sqrt{(\cos\alpha \cdot \delta l)^2 + (l \cdot (-\sin\alpha) \cdot \delta\alpha)^2} = \sqrt{(0.94 \cdot 0.1)^2 + (10 \cdot [-0.34] \cdot 0.05)^2} = 0.2 \text{ m}$$

$$h = 9.4 \pm 0.2 \text{ m}$$

From Yagil

Propagating Errors for Experiment 1

$$\rho = \frac{3}{4\pi}\frac{g}{GR_e}$$

Formula for density.

$$\sigma_\rho = \frac{3}{4\pi}\frac{1}{GR_e}\sigma_g \oplus \frac{-3}{4\pi}\frac{g}{GR_e^2}\sigma_{R_e}$$

Take partial derivatives and add errors in quadrature

Or, in terms of relative uncertainties: $\frac{\sigma_\rho}{\rho} = \frac{\sigma_g}{g} \oplus \frac{\sigma_{R_e}}{R_e}$

shorthand notation for quadratic sum: $\sqrt{a^2 + b^2} = a \oplus b$

From Yagil

Propagating Errors for R_e

$$R_e = \frac{2C}{\omega^2}\left(\frac{\sqrt{h_1}-\sqrt{h_2}}{\Delta t}\right)^2$$

basic formula

$$\sigma_{R_e} = \frac{\partial R_e}{\partial \Delta t}\sigma_{\Delta t} \oplus \frac{\partial R_e}{\partial h_1}\sigma_{h_1} \oplus \frac{\partial R_e}{\partial h_2}\sigma_{h_2}$$

Propagate errors (use shorthand for addition in quadrature)

$$\sigma_{R_e} = \frac{2R_e}{\Delta t}\sigma_{\Delta t} \oplus \frac{R_e}{\sqrt{h_1}\left(\sqrt{h_1}-\sqrt{h_2}\right)}\sigma_{h_1} \oplus \frac{R_e}{\sqrt{h_2}\left(\sqrt{h_1}-\sqrt{h_2}\right)}\sigma_{h_2}$$

Note that the error blows up at $h_1=h_2$ and at $h_2=0$.

From Yagil

Remember

- Finish Lab #1
- Read lab description, prepare for Lab #2 for Thursday
- Read Taylor through Chapter 5
- Problems 5.2, 5.36