Physics 2BL: Homework Set 02

Taylor Problems: 4.6, 4.14, 4.18, 4.26

4.6

(a) N={10,13,8,15,13,14,13,19,8,13,13,7,8,6,8,11,12,8,7}

MEAN VALUE: $\overline{N} = \sum_i N_i$ / (#DATA POINTS)

$$\overline{N} = \frac{1}{20}[10+13+8+15+8+13+14+13+19+8+13+13+7+8+6+8+11+12+8+7]$$

$$\overline{N} = 10.7$$

STD DEVIATION: $\sigma_N = \sqrt{\frac{1}{(\#DATA\_PTS-1)}\sum_i (N_i - \overline{N})^2}$

$$\sigma_N = \sqrt{\begin{array}{l}(\tfrac{1}{19})[(10-10.7)^2 + 5(13-10.7)^2 + 6(8-10.7)^2 + (15-10.7)^2 + (14-10.7)^2 \\ + (19-10.7)^2 + 2(7-10.7)^2 + (6-10.7)^2 + (11-10.7)^2 + (12-10.7)^2\end{array}}$$

$$= \{(\frac{1}{19})[(0.7)^2 + 5(2.3)^2 + 6(2.7)^2 + (4.3)^2 + (3.3)^2 + (8.3)^2 + 2(3.7)^2 + (4.7)^2 + (0.3)^2 + (1.3)^2]\}^{\frac{1}{2}}$$

$$\sigma_N \approx 3.4$$

(b) $(\sigma_N)^2 \approx 11.6, \overline{N} \approx 10.7$

SO, AS EXPECTED $\sigma_N \approx \sqrt{\overline{N}}$

4.14

(a) SEE ABOVE PROBLEM 4.6

(b) 68.27% OF THE DATA SHOULD FALL WITHIN THE RANGE $\overline{N} \pm \sigma_n \Rightarrow 31.73\%$ OF THE DATA SHOULD LIE OUTSIDE

SINCE WE TOOK 20 DATA POINTS, WE SHOULD EXPECT THAT (20)(31.73%) = 6.3 (ROUND TO 6) DATA POINTS LIE OUTSIDE THIS RANGE

$7.3 \le N_i \le 14.1$

EXPECT 6 DATA POINTS

COUNT 5 DATA POINTS

(c) 95% OF THE DATA SHOULD FALL WITHIN THE RANGE $\overline{N} \pm 2\sigma_n \Rightarrow 5\%$ OF THE DATA POINTS SHOULD LIE OUTSIDE. SINCE WE TOOK 20 DATA POINTS, WE SHOULD EXPECT THAT (20)(5%) = 1 DATA POINT LIES OUTSIDE OF THIS RANGE $3.9 \le N_i \le 17.5$

EXPECT ONE DATA POINT

COUNT ONE DATA POINT

4.18

$\sigma_{\bar{u}} = \sigma_u / \sqrt{N} \Rightarrow N = (\sigma_u / \sigma_{\bar{u}})^2$

$\sigma_u = 10 ms^{-1}$

(a) TO OBTAIN $\sigma_{\bar{u}} = \pm 3 ms^{-1}$

$N = (10 ms^{-1} / 3 ms^{-1})^2 = (10/3)^2 = 11.1$ ROUND

$N \approx 11$ MEASUREMENTS

(b) TO OBTAIN $\sigma_{\bar{u}} = \pm 0.5 ms^{-1}$

$N = (10 ms^{-1} / 0.5 ms^{-1})^2 = (20)^2 = 400$

$N \approx 400$ MEASUREMENTS

4.26

$R = V / I$

VOLTAGE, V (VOLTS)	11.2	13.4	15.1	17.7
CURRENT, I (AMPS)	4.67	5.46	6.28	7.22

(a)

$R_1 = V_1 / I_1 = (11.2V)/(4.67A) = 2.40\Omega$

$R_2 = V_2 / I_2 = (13.4V)/(5.46A) = 2.45\Omega$

$R_3 = V_3 / I_3 = (15.1V)/(6.28A) = 2.40\Omega$

$R_4 = V_4 / I_4 = (17.7V)/(7.22A) = 2.45\Omega$

$\bar{R} = 2.425\Omega$

$\delta R_{RANDOM} = \sigma_{\bar{R}} = \sigma_R / \sqrt{N}$

$\sigma_R = \{\frac{1}{3}[(2.425\Omega - 2.40\Omega)^2 + (2.425\Omega - 2.40\Omega)^2 + (2.425\Omega - 2.45\Omega)^2 + (2.425\Omega - 2.45\Omega)^2]\}^{1/2}$

$\sigma_R = 0.028867513\Omega$

$\Rightarrow \delta R_{RANDOM} = \sigma_R / \sqrt{4} \approx 0.014\Omega$

$\bar{R} \pm \delta R_{RANDOM} = 2.425\Omega \pm 0.014\Omega$

(b)

$\frac{\delta R_{SYS}}{R} = \sqrt{\left(\frac{\delta V}{V}\right)^2 + \left(\frac{\delta I}{I}\right)^2} = \sqrt{(0.02)^2 + (0.02)^2}$

$\Rightarrow \delta R_{SYS} = (\bar{R})(0.028) \approx 0.07\Omega$

$\delta R = \sqrt{(\delta R_{RAN})^2 + (\delta R_{SYS})^2} \approx 0.07\Omega$

IF $\bar{R}$ IS ROUNDED TO 2.43Ω, THE FINAL RESULT IS $\bar{R} \pm \delta R_{TOTAL} = 2.43\Omega \pm 0.07\Omega$.
THIS VALUE IS CONSISTENT WITH THE GIVEN VALUE OF $R_{RATED} = 2.50\Omega$.