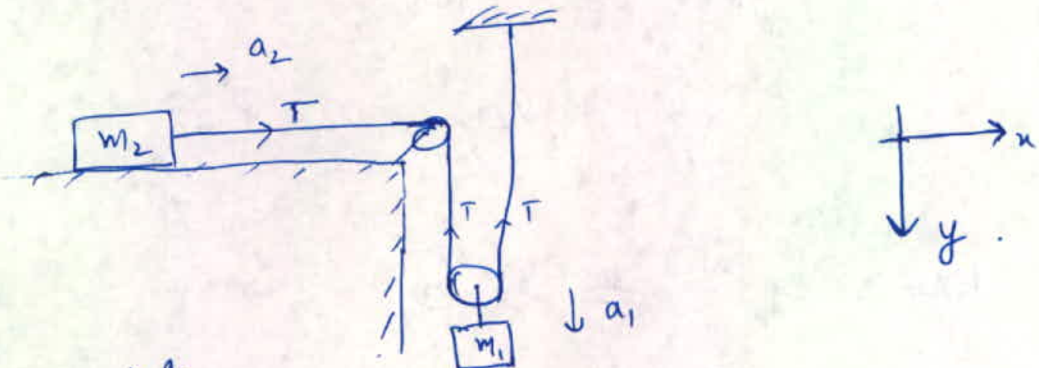
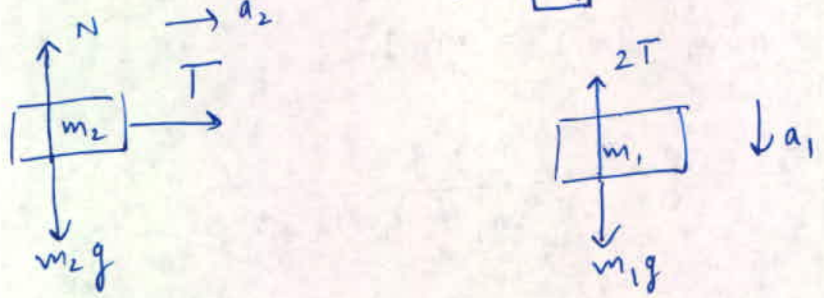
HW # 3

PHYS 4A
WINTER '15

CHAPTER - 6

16.)





For m_2

$N = m_2 g$.

& $T = m_2 a_2$ — ①

For m_1

$m_1 g - 2T = m_1 a_1$ — ②

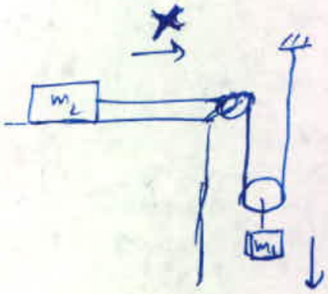
From equations ① & ②.

$$m_1 g - 2(m_2 a_2) = m_1 a_1$$

$$\Rightarrow m_1 a_1 + 2 m_2 a_2 = m_1 g \quad — ③$$

Also, $a_2 = 2 a_1$ — ④ { Due to the constraint of rope }

i.e.



when Block m_2 moves x distance towards right, Block m_1 should fall $x/2$ distance

$\therefore$ $y = x/2$

from $Y = X/2$

If you differentiate it twice, you will get.

$$a_1 = a_2/2 \quad \text{or} \quad a_2 = 2a_1$$

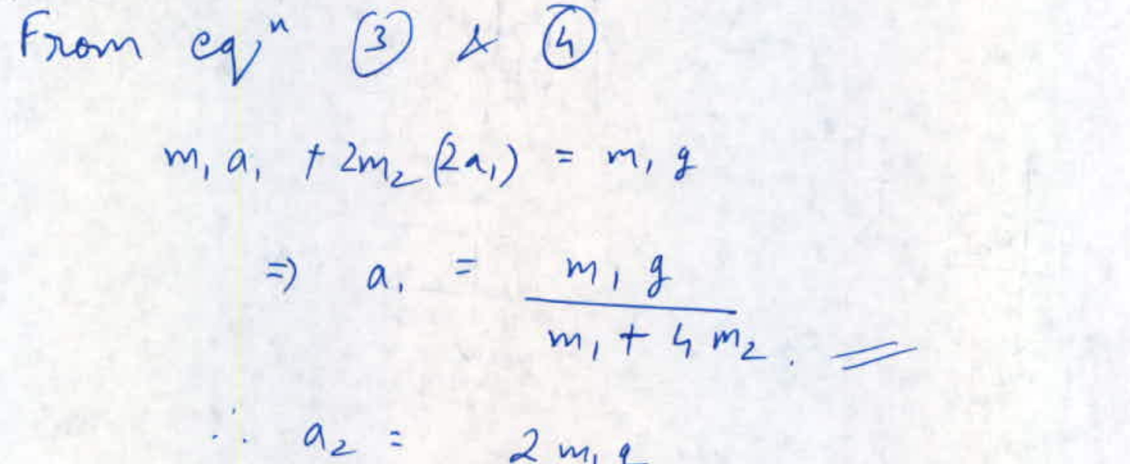
From eq^n (3) & (4)

$$m_1 a_1 + 2m_2(2a_1) = m_1 g$$

$$\Rightarrow a_1 = \frac{m_1 g}{m_1 + 4m_2}$$

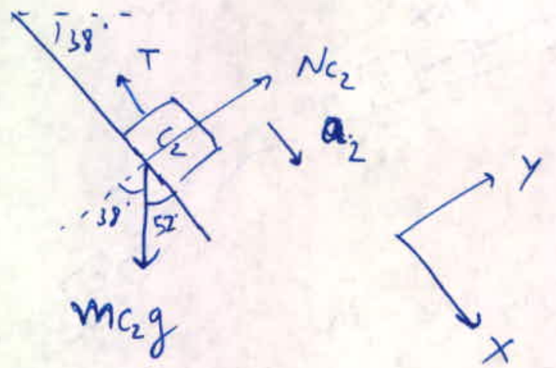
$$\therefore a_2 = \frac{2m_1 g}{m_1 + 4m_2}$$

20.) a) C_1 & $C_2 \rightarrow$ 2 climbers



$$T + m_{c_1} g \cos(78°) = m_{c_1} a_1 \quad \rightarrow X\text{-direction} \quad -(1)$$

& $$N_{c_1} = m_{c_1} g \cos(12°) \quad \rightarrow Y\text{-direction}$$



$$\therefore\ -T + m_{c_2} g \cos(52^\circ) = m_{c_2} a_2 \quad - \ (2)$$

& $N_{c_2} = m_{c_2} g \cos(38^\circ)$

Also,

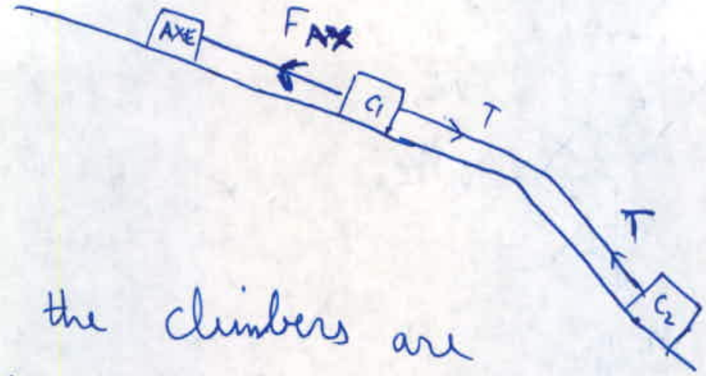
$a_1 = a_2$ — (3) { due to constraint of rope }

From (1), (2) & (3) { (1) + (2) and using $a_1 = a_2$ }

$$m_{c_1} g \cos(78^\circ) + m_{c_2} g \cos(52^\circ) = (m_{c_1} + m_{c_2}) a_1$$

$$\Rightarrow a_1 = 3.86 \ m/s^2$$

b)



Now, the climbers are at rest.

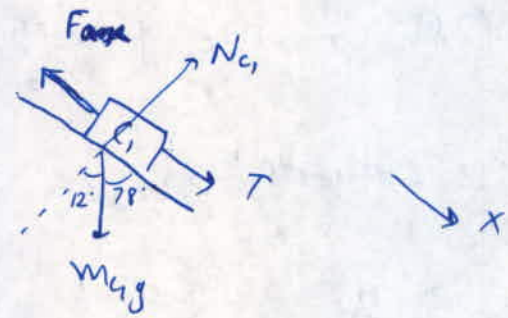
$\therefore$ Tension in the rope will change as acceleration is Zero. $\{a_2 = a_1 = 0\}$

For C_2, Equations remains the same.

$\therefore$ Put $a_2 = 0$ in equation (2)

then $T = m_{c_2}\, g \cos(52^\circ)$ — (4)

For C_1,

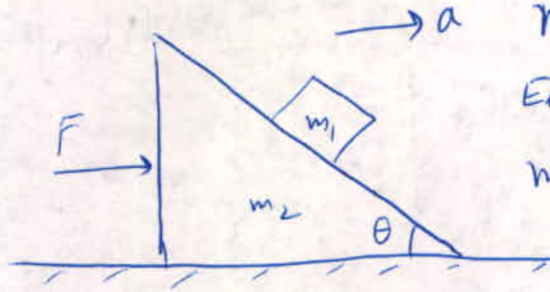


$$\therefore \quad -F_{ax} + T + m_{c_1}\, g \cos(78^\circ) = m(0) = 0$$

$$\Rightarrow F_{ax} = m_{c_1}\, g \cos(78^\circ) + m_{c_2}\, g \cos(52^\circ) \quad \{\text{from eq}^n (4)\}$$

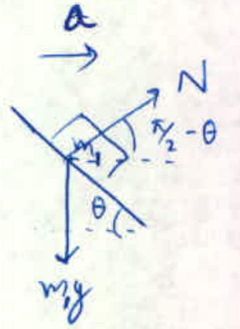
$$\Rightarrow F_{ax} = 532.9 \text{ N}$$

22.)



m_1 not sliding is EQUAL to m_1 & m_2 moving with same acceleration in horizontal direction

For m_1

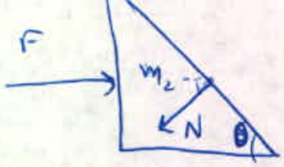


$$N\cos\left(\tfrac{\pi}{2}-\theta\right) = m_1 a \quad (x\text{-direction})$$

$$\Rightarrow N\sin\theta = m_1 a. \quad -①$$

$$\& \quad N\cos\theta = m_1 g \quad -②$$

For m_2



$$F - N\sin\theta = m_2 a$$

$$\Rightarrow F - m_1 a = m_2 a \quad \{\text{from eq}^n ①\}$$

$$\Rightarrow F = (m_1 + m_2)\, a$$

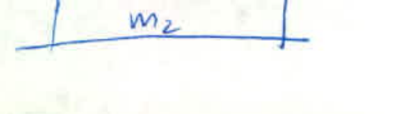
Also, from eq^n ① & ②

$$\tan\theta = a/g.$$

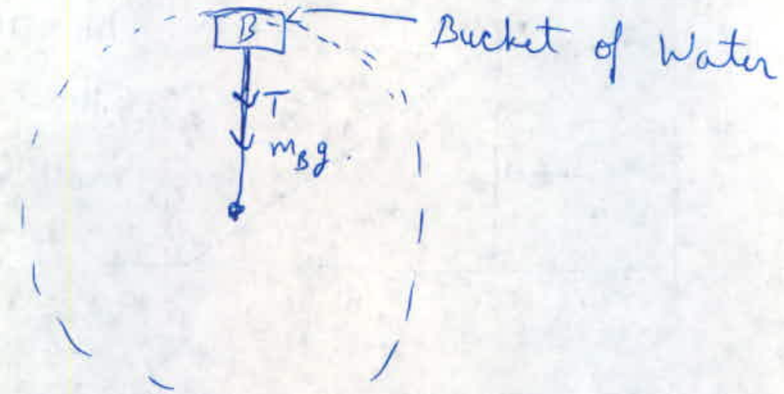
$$\Rightarrow a = g\tan\theta.$$

$$\therefore F = (m_1 + m_2)\, g\tan\theta$$

Check: If $\theta = 0$, then $F = 0$, which is indeed true as no Force is required then



36.)



At topmost point,

$$T + m_B g = m_B \frac{v^2}{r}.$$

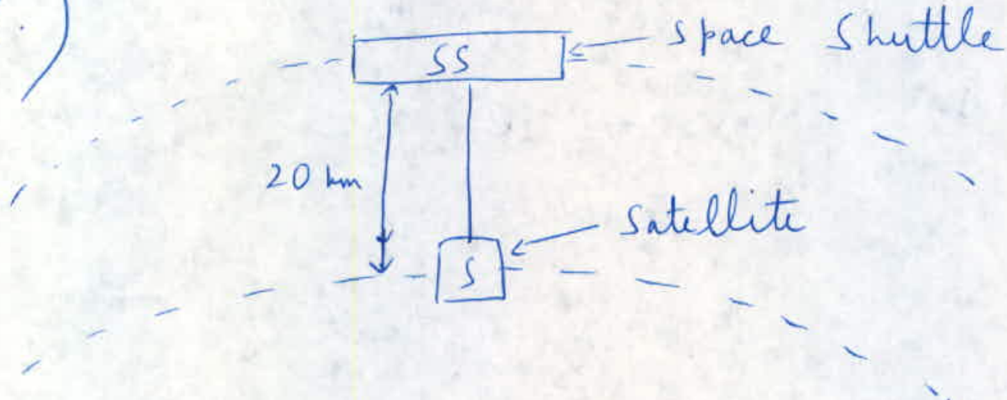
But for minimum speed, $T = 0$.

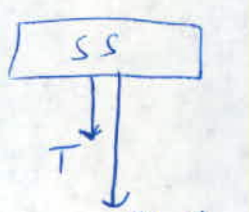
$$\therefore \cancel{m_B} g = \cancel{m_B} \frac{v^2}{r}$$

$$\Rightarrow v = \sqrt{rg} = \sqrt{0.85 \times 9.8} \text{ m/s}$$

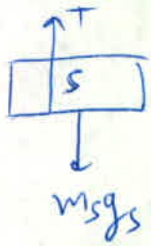
$$\Rightarrow v_{min} = 2.89 \text{ m/s}$$

38.)





$$\therefore T + m_{ss} g_{ss} = m_{ss} \frac{v_{ss}^2}{r_{ss}} \quad -(1)$$



$$-T + m_s g_s = m_s \frac{V_s^2}{r_s} \quad -(2).$$

Now, since $\boxed{SS}$ & $\boxed{S}$ move together,
∴ their Angular velocity is same,
i.e. they sweep equal angle per unit of time.

& $\omega = V/r$.
Angular velocity

Eq^n (1) → $T + m_{ss} g_{ss} = m_{ss} \omega^2 r_{ss} \quad -(3)$

Eq^n (2) → $-T + m_s g_s = m_s \omega^2 r_s \quad -(4).$

ω is same

(3) + (4).

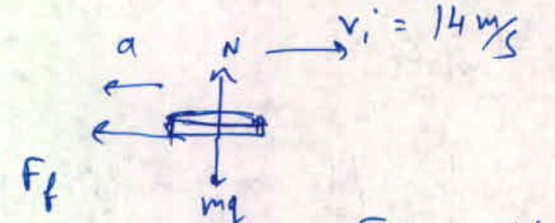
$$\omega^2 = \frac{(m_{ss} g_{ss} + m_s g_s)}{(m_{ss} r_{ss} + m_s r_s)}$$

Substituting this in eq^n (3)

$$T = \frac{m_{ss} r_{ss} (m_{ss} g_{ss} + m_s g_s)}{(m_{ss} r_{ss} + m_s r_s)} - m_{ss} g_{ss}$$

You need m_{ss} also. Everything else - r_{ss}, r_s, g_{ss}, g_s, m_s is given in the Problem.

42.)



$$F_f = \mu_k N = ma.$$

$$\& \quad N = mg$$

$$\therefore \quad \mu_k (mg) = ma$$

$$\Rightarrow \quad a = \mu_k g \rightarrow \text{in negative } x\text{-direction}$$

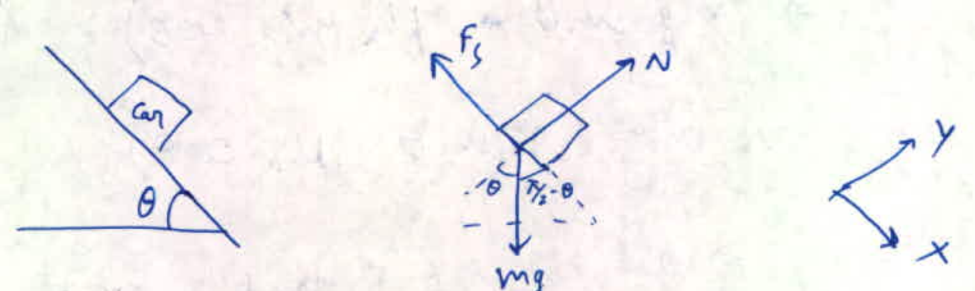
$$\therefore \quad V_f^2 = 0 = V_i^2 + 2(a)x$$

$$\Rightarrow \quad a = \frac{V_i^2}{2x}$$

$$\Rightarrow \quad \mu_k = \frac{V_i^2}{2gx}$$ 0.17

$$\Rightarrow \quad \mu_k = 0.18$$

46.) a)



$$\therefore \quad N = mg\cos\theta \rightarrow y\text{-direction}$$

$$\& \quad F_s = \mu_s N = mg\sin\theta \rightarrow x\text{-direction}$$

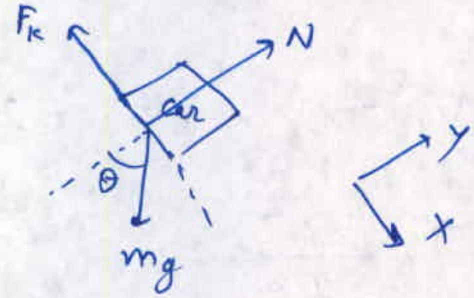
$$\Rightarrow \quad \mu_s \cancel{mg}\cos\theta = \cancel{mg}\sin\theta$$

$$\Rightarrow \mu_s = \tan\theta$$

$$\Rightarrow \theta = \tan^{-1}\mu_s = \tan^{-1}(0.14)$$

$$\Rightarrow \theta = 7.97^\circ$$

b) When on a slope just steeper than this, kinetic Friction will act on the car instead of Static Friction



$$N = mg\cos\theta$$

$$\& \quad mg\sin\theta - F_k = ma.$$

$$\Rightarrow \cancel{m}g\sin\theta - \mu_k(\cancel{m}g\cos\theta) = \cancel{m}a$$

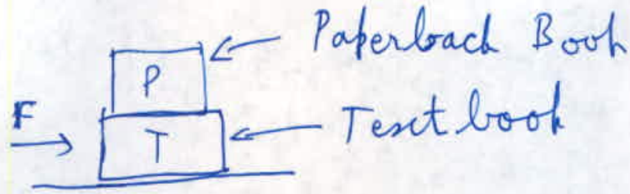
$$\Rightarrow a = g(\sin\theta - \mu_k\cos\theta)$$

$$\Rightarrow a = g\cos\theta\,(\tan\theta - \mu_k)$$

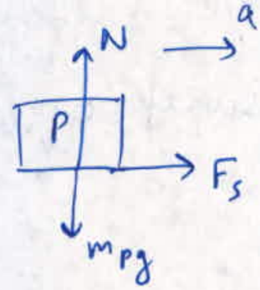
$$\Rightarrow a = g\cos\theta\,(\mu_s - \mu_k) \qquad \left\{\begin{array}{l}\text{Put } \mu_s = \tan\theta \\ \text{i.e. } \theta = 7.97^\circ\end{array}\right\}$$

$$\Rightarrow a = 0.505\ m/s^2$$

52.)



For the initial part when they accelerate together



$$m_p a = F_s = \mu_s N = \mu_s m_p g$$

$$\Rightarrow a = \mu_s g$$

and $$v_f = v_i + at$$

$$\Rightarrow 0.96 = 0 + (\mu_s g)(0.42)$$

$$\Rightarrow \mu_s = 0.233$$

For the next part,

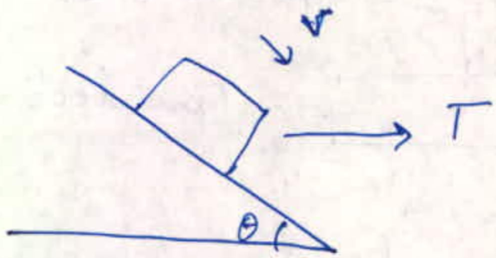
$$a = \frac{0.96 \text{ m/s}}{0.33 \text{ s}} = 2.91 \text{ m/s}^2$$

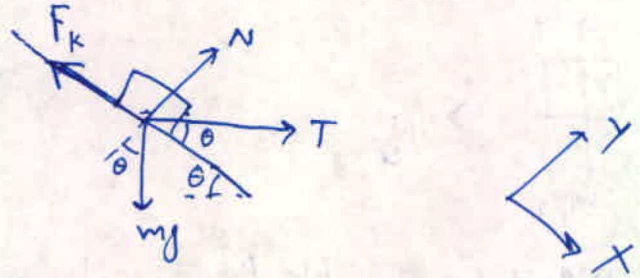
If there would have been static friction,

then $$\mu_s = a/g = 0.297$$

$$\therefore 0.233 < \mu_s < 0.297$$

60.)





in y- direction

$$T\sin\theta + N = mg\cos\theta \qquad -(1)$$

in x - direction

$$mg\sin\theta + T\cos\theta - F_k = ma = 0. \qquad -(2)$$

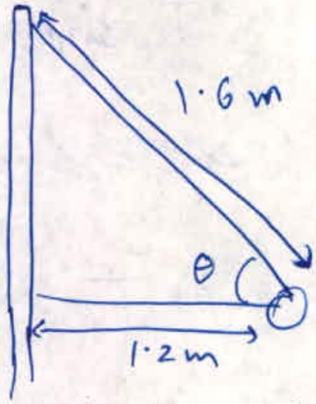
$$F_k = \mu_k N = \mu_k\left(mg\cos\theta - T\sin\theta\right) \quad \left\{\text{From eq}^n\ (1)\right\}$$

Put this in eq^n (2).

$$\therefore\ mg\sin\theta + T\cos\theta - \mu_k\left(mg\cos\theta - T\sin\theta\right) = 0$$

$$\therefore\ T = \frac{mg\left(\mu_k\cos\theta - \sin\theta\right)}{\left(\cos\theta + \mu_k\sin\theta\right)}$$

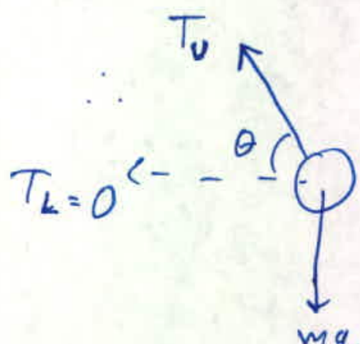
76.)



$\tan\theta = \frac{1.6}{1.2} = \frac{4}{3}$

$\theta = 53.13°$

a) For lower string to be just taut, radius of circular motion of ball should be 1·2 m and Tension in the lower string will be 0.



$\therefore\ T\sin\theta = mg \qquad -①$

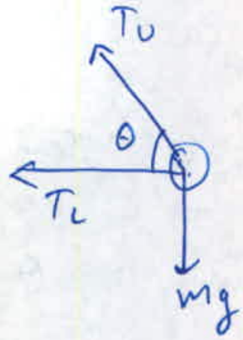
$\&\ T\cos\theta = \frac{m V_{min}^2}{r} \qquad -②$

$\therefore\ \frac{①}{②} \Rightarrow \tan\theta = \frac{g}{\left(\frac{V_{min}^2}{r}\right)} = \frac{4}{3}$

$\Rightarrow V_{min}^2 = \frac{3gr}{4}$

$\Rightarrow V_{min} = 2.97\ m/s$

b)



$$T_U \sin\theta = mg.$$

$$\cancel{\Rightarrow T_U = \frac{mg}{\cos(53.13)} =}$$

$$\Rightarrow T_U = \frac{mg}{\sin(53.13^\circ)} = 10.29\ N$$

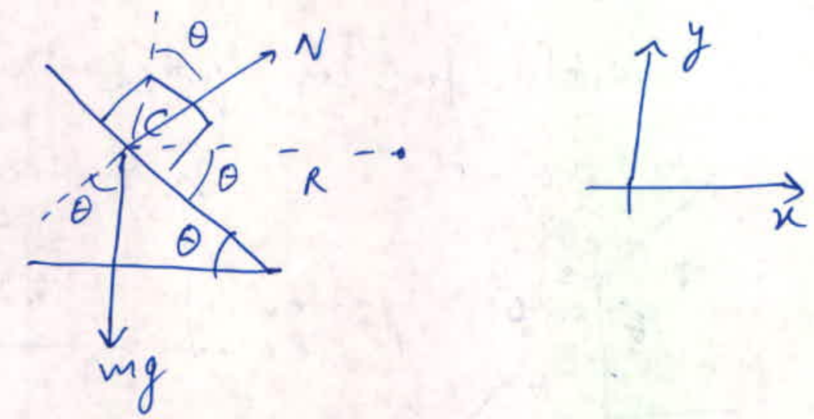
And.

$$T_L + T_U \cos\theta = \frac{m v^2}{r}.$$

$$\Rightarrow T_L = \frac{0.84 \times 5^2}{1.2} - 10.29 \cos(53.13^\circ)$$

$$\Rightarrow T_L = 11.33\ N$$

81.) Given: Banked Highway Turn of Radius R is designed for speed V_d.



Since, Car is doing circular motion with radius R along the Horizontal, it is IMPORTANT to define our axes along and perpendicular to horizontal. Because the Net Acceleration of car is $\frac{V_d^2}{R}$ along the horizontal direction.

∴ In y-direction,

$$N\cos\theta = mg \quad -①$$

In x-direction

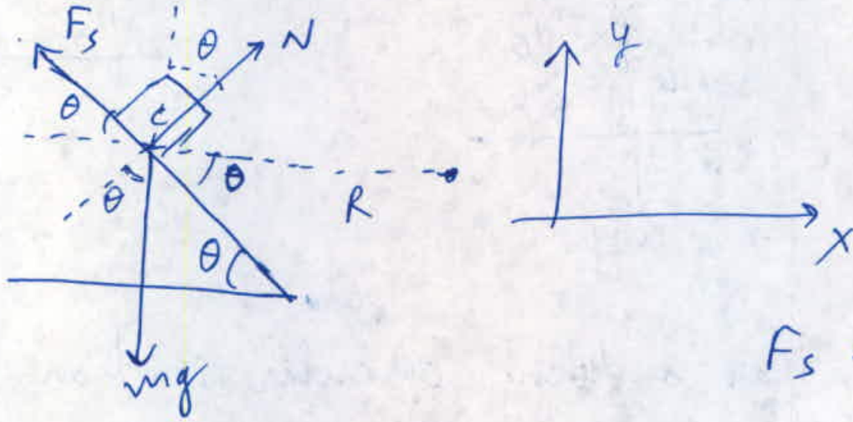
$$N\sin\theta = \frac{mV_d^2}{R} \quad -②$$

From equations ① & ②

$$\tan\theta = \frac{V_d^2}{gR} \quad -③$$

Now,

With friction force and $V = V_d + \Delta V$



$$F_s = \mu_s N$$

In y-direction

$$N\cos\theta + F_s \sin\theta = mg$$

$$\Rightarrow N\cos\theta + \mu_s N \sin\theta = mg$$

$$\Rightarrow N(\cos\theta + \mu_s \sin\theta) = mg. \quad -④$$

In x-direction

$$N\sin\theta - F_s\cos\theta = \frac{mv^2}{R} \quad \{v^2 = (V_d + \Delta v)^2\}$$

$$\Rightarrow N\sin\theta - \mu_s N\cos\theta = \frac{mv^2}{R}.$$

$$\Rightarrow N(\sin\theta - \mu_s\cos\theta) = \frac{mv^2}{R} \quad -⑤$$

From Equations (4) & (5)

$$\frac{(4)}{(5)} \rightarrow \frac{\cancel{N}(\cos\theta + \mu_s \sin\theta)}{\cancel{N}(\sin\theta - \mu_s \cos\theta)} = \frac{\cancel{m}g}{\cancel{m}v^2/R}$$

$$\Rightarrow \frac{\cancel{\cos\theta}(1 + \mu_s \tan\theta)}{\cancel{\cos\theta}(\tan\theta - \mu_s)} = \frac{gR}{v^2}$$

$$\Rightarrow v^2 + v^2\mu_s \tan\theta = gR\tan\theta - \mu_s gR.$$

$$\Rightarrow \mu_s(gR + v^2\tan\theta) = gR\tan\theta - v^2$$

From eq^n (3) $\tan\theta = \dfrac{V_d^2}{gR}$

$$\therefore \mu_s\left(gR + \frac{v^2 V_d^2}{gR}\right) = \cancel{gR}\frac{V_d^2}{\cancel{gR}} - v^2$$

and $v = V_d + \Delta V$

$$\Rightarrow \mu_s = \frac{-\Delta V^2 - 2V_d\Delta V}{gR + \dfrac{v^2 V_d^2}{gR}}$$

$$\Rightarrow \mu_s = \frac{|\Delta V|}{gR}\,\frac{(\Delta V + 2V_d)}{\left[1 + \left(\dfrac{V V_d}{gR}\right)^2\right]}$$