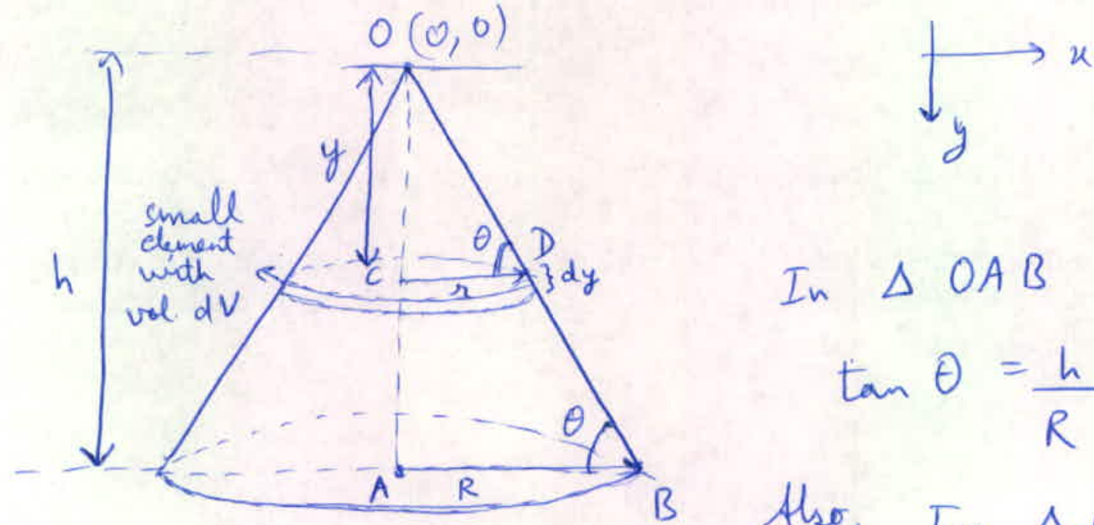
HW # 5

PHYS 4A

WINTER '15

Chapter - 10

18.)



In Δ OAB

$$\tan\theta = \frac{h}{R}. \quad -①$$

Also, In Δ OCD

$$\tan\theta = y/r \quad -②$$

From ① & ②

$$y/r = \frac{h}{R}$$

$$\Rightarrow r = \frac{R}{h} y \quad -③$$

$$X_{CM} = 0 \quad \text{(by symmetry)}$$

$$Y_{CM} = \frac{1}{M}\int_0^h y\, dm$$

$$dm = \frac{M}{V}\, dV \quad \text{(see example 10-3 in book)}$$

$$\& \ \frac{M}{V} = \rho$$

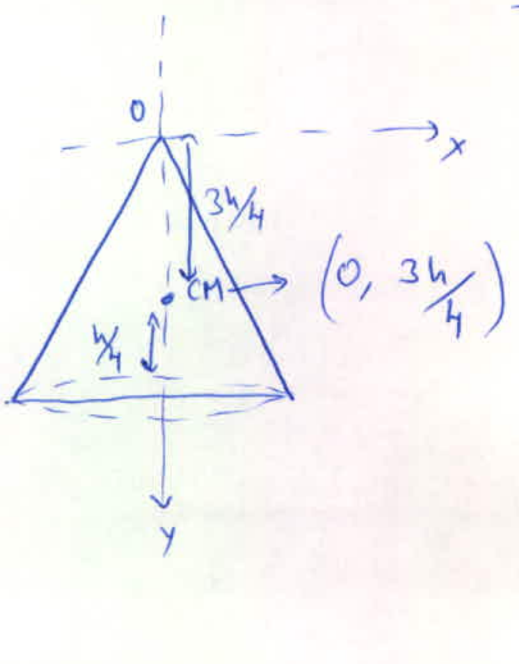
$$dV = \pi r^2 \, dy$$
$$= \frac{\pi R^2}{h^2} y^2 \, dy \quad \text{(from eq (3))}$$

$$\therefore \quad dm = \rho \, dV = \rho \frac{\pi R^2}{h^2} y^2 \, dy$$

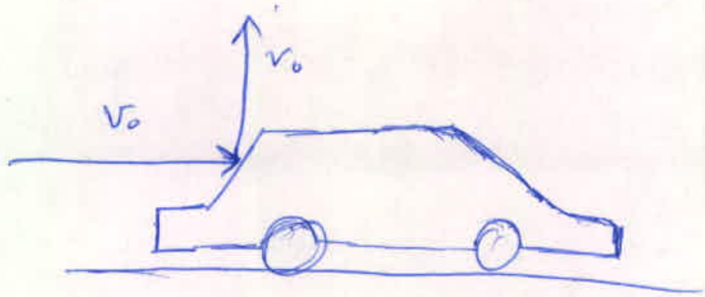
$$y_{cm} = \frac{1}{M} \int_0^h y \, dm$$
$$= \frac{\rho \pi R^2}{h^2 M} \int_0^h y^3 \, dy$$
$$= \frac{\rho \pi R^2 h^2}{4M}$$

$$\rho = M/V = \frac{M}{(\pi R^2 h/3)}$$

$$\therefore \quad y_{cm} = \frac{M}{(\pi R^2 h)/3} \times \frac{\pi R^2 h^2}{4M}$$
$$= \frac{3h}{4}$$



32.)



a) $$\vec{F}_{\text{net ext on car}} = \frac{d\vec{P}}{dt} = M_{car}\frac{d\vec{V}_{car}}{dt} \qquad -①$$

$$\text{Total } \vec{P}_i = \cancelto{0}{M\vec{V}_{car_i}} + \Delta m\, v_0\, \hat{i}$$

$$\text{Total } \vec{P}_f = M\vec{V}_{car_f} + \Delta m\, v_0\, \hat{j}$$

Conserving Total momentum,

$$\vec{P}_i = \vec{P}_f$$

$$\Delta m\, v_0\, \hat{i} = M\vec{V}_{car_f} + \Delta m\, v_0\, \hat{j}$$

$$\Rightarrow M\left(\frac{\Delta\vec{V}_{car}}{\Delta t}\right) = \frac{\Delta m}{\Delta t}\left(v_0\hat{i} - v_0\hat{j}\right) \quad \left\{\begin{array}{l}\text{divide} \\ \text{both sides} \\ \text{by } \Delta t\end{array}\right\}$$

$$\Rightarrow M\frac{d\vec{V}_{car}}{dt} = \frac{dm}{dt}v_0\left(\hat{i} - \hat{j}\right)$$

$$\Rightarrow \vec{a}_{car} = \frac{1}{M}\frac{dm}{dt}v_0\left(\hat{i} - \hat{j}\right)$$

$$\left|\vec{a}_{car}\right| = \frac{1}{M}\frac{dm}{dt}\frac{v_0}{\sqrt{2}}$$

b) $\underline{\underline{V_0}}$.

Because then relative velocity between car ~~t~~ and the water stream will be 0. The jet of water will not exert any force then.

38.) $v_f = v_i + v_{ex} \ln\left(\frac{M_i}{M_f}\right)$ (Equation 10-11) in Book.

$v_i = 0$

$V_{ex} = 2.5 \text{ km/s}$

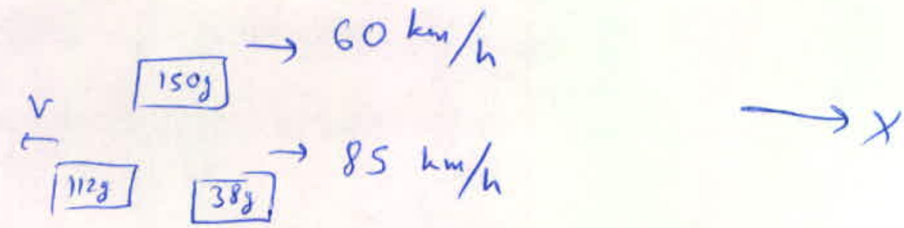
$M_i = M$

$M_f = M - 80\% \text{ of } M = 0.2M$

$\therefore \quad v_f = 2.5 \ln\left(\frac{\cancel{M}}{0.2\cancel{M}}\right) \text{ km/s}$

$\Rightarrow v_f = 4.02 \text{ km/s}$

42.)



Conserving Momentum,

$$\frac{150}{1000} \times 60 = \frac{38}{1000} \times 85 - \frac{112}{1000} v$$

$\Rightarrow \quad v = -51.52$ km/h

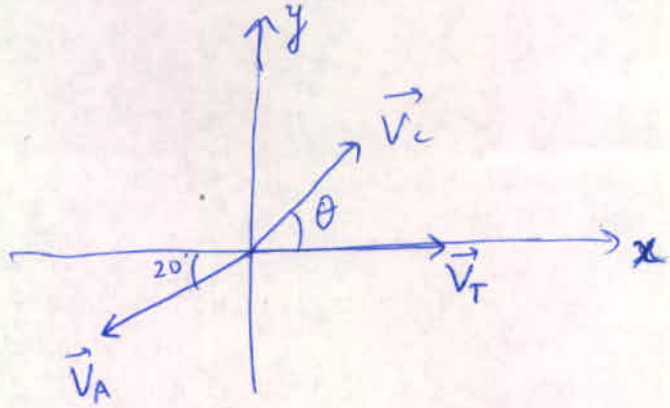
-ve implies 112g also goes straight ahead

$\therefore$ Gain in energy $= K_f - K_i$

$$= \frac{1}{2} m_{38} V_{38}^2 + \frac{1}{2} m_{112} V_{112}^2 - \frac{1}{2} m_{150} V_{150}^2$$

$$\simeq 1 \text{ J}$$

50.)



Conserving Momentum in X & Y-directions

$$X \rightarrow \quad m_c V_c \cos\theta + m_T|\vec{V}_T| - m_A|\vec{V}_A|\cos 20^\circ = \underset{P_i = 0}{\underset{\uparrow}{0}}$$

$$\Rightarrow \quad V_c \cos\theta = \frac{1}{m_c}\left(m_A|\vec{V}_A|\cos 20^\circ - m_T|\vec{V}_T|\right) \quad -①$$

$$Y \rightarrow \quad m_c V_c \sin\theta - m_A|\vec{V}_A|\sin 20^\circ = 0$$

$$\Rightarrow \quad V_c \sin\theta = \frac{m_A|\vec{V}_A|\sin 20^\circ}{m_c} \quad -②$$

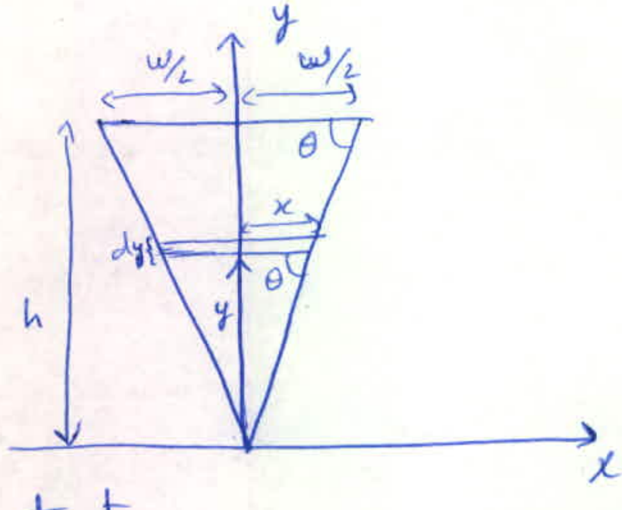
$$\frac{②}{①} \rightarrow \quad \tan\theta = \frac{m_A|\vec{V}_A|\sin 20}{\left(m_A|\vec{V}_A|\cos 20^\circ - m_T|\vec{V}_T|\right)}$$

$$\Rightarrow \quad \theta = 34.35^\circ$$

$$\therefore \quad V_c = \frac{m_A|\vec{V}_A|\sin 20^\circ}{m_c \sin(34.35^\circ)} \quad (\text{from eq}^n ②)$$

$$= 5.33\ m/s$$

60.) X_{CM} is 0 by symmetry.



Given: $\rho \propto y^\alpha$

or $\rho = c y^\alpha$

where c is some constant.

$$\tan\theta = \frac{h}{(w/2)} = y/x$$

$$\therefore \; x = \frac{y}{h}\, w/2 \qquad -①$$

Now,

$$Y_{CM} = \frac{1}{M}\int y \, dm$$

where $dm = \rho \, dA$

and $dA = 2x\,dy$

$$= \cancel{2}\left(\frac{y}{h}\,\frac{w}{\cancel{2}}\right) dy \qquad \text{(from eq ①)}.$$

$$= \frac{w}{h}\, y\, dy$$

$$\therefore \; dm = \rho \, dA$$

$$= c y^\alpha \, w/h \, y \, dy$$

$$= c\,w/h \; y^{\alpha+1}\, dy. \qquad -②$$

$$\therefore\ Y_{cm} = \frac{1}{M}\int y\,dm$$

$$= \frac{1}{M}\int_0^h y\left(\frac{cw}{h}\right)y^{\alpha+1}\,dy$$

$$= \frac{1}{M}\left(\frac{cw}{h}\right)\left.\frac{y^{\alpha+3}}{\alpha+3}\right|_0^h$$

$$= \frac{1}{M}\left(\frac{cw}{h}\right)\left(\frac{h^{\alpha+3}}{\alpha+3}\right) \quad -\ (3)$$

Now,

$$M = \int dm$$

$$= \int \rho\,dA$$

$$= \int_0^h c y^{\alpha}\,\frac{w}{h}\,y\,dA \quad (\text{from eq (2)})$$

$$= \frac{cw}{h}\left.\frac{y^{\alpha+2}}{\alpha+2}\right|_0^h$$

$$= \left(\frac{cw}{h}\right)\frac{h^{\alpha+2}}{\alpha+2}. \quad -\ (4)$$

From eq^n (3) & (4)

$$Y_{cM} = \cancel{\left(\frac{cw}{h}\right)}\frac{h^{\alpha+3}}{(\alpha+3)} \times \cancel{\left(\frac{h}{cw}\right)}\frac{\alpha+2}{h^{\alpha+2}}$$

$$= \frac{(\alpha+2)h}{\alpha+3}.$$

Chapter - 11

6.) a) $\vec{I} = \vec{F}\Delta t = \Delta \vec{p}$

$$\Rightarrow (42\,\hat{i} + 17\hat{j})\,\mu N \times 0.12\, fs$$

$$= m_p \left(\vec{V}_f - 4.3\, Mm/s\, \hat{i}\right)$$

$$\Rightarrow \vec{V}_f - 4.3\, Mm/s\, \hat{i} = 0.0717\,(42\,\hat{i} + 17\hat{j})\, Mm/s$$

$$\Rightarrow \vec{V}_f = (7.31\,\hat{i} + 1.22\,\hat{j})\, Mm/s$$

b) $\theta = \tan^{-1}\left(\frac{V_y}{V_x}\right) = \tan^{-1}\left(\frac{1.22}{7.31}\right)$

$$\Rightarrow \theta = 9.48^{\circ}$$

18.)

Momentum Conservation -

$$m v = (m + m)\, V_f$$

$$\Rightarrow v_f = v/2$$

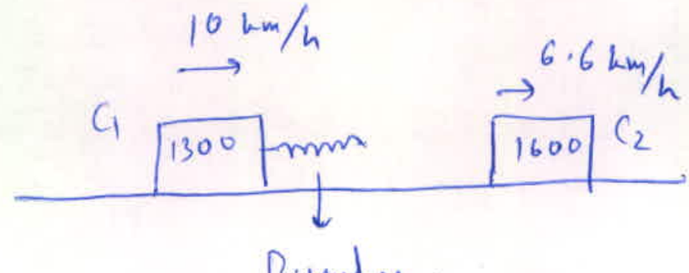
$$K_i = \frac{1}{2} m v^2$$

$$K_f = \frac{1}{2} (m+m)\left(\frac{v}{2}\right)^2 = \frac{1}{4} m v^2$$

i.e. $K_f = \frac{1}{2} K_i$

$\therefore$ Half the initial Kinetic Energy is lost.

30.)



At the maximum compression of spring, the relative velocity of cars will be zero.

i.e. $\vec{V}_{c_1/c_2} = 0$

$\Rightarrow V_{c_1} = V_{c_2}$.

{ This is because if $V_{c_1} > V_{c_2}$ then the spring can/will still compress. And if $V_{c_1} < V_{c_2}$ then the cars are moving apart, implying that point of maximum compression of spring has already passed. }

Conserving momentum between these 2 points,

$$\vec{P}_i = \vec{P}_f$$

$$\Rightarrow (1300 \times 10) + (1600 \times 6.6) = 1300 \times V_c + 1600 \times V_c$$

Both cars have same velocity

$$\Rightarrow V_c = 8.12 \text{ m/s}$$

Since there is no external force and no loss of energy,

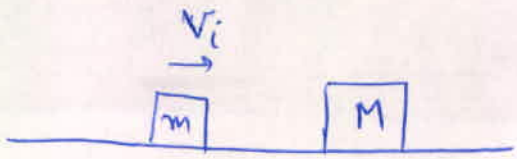
$$\therefore \Delta U + \Delta K = W_{nc} = 0$$

$$\Rightarrow \underbrace{\tfrac{1}{2} k x^2}_{U_f} - \underbrace{0}_{U_i} + \underbrace{\tfrac{1}{2}(1300+1600)V_c^2}_{K_f} - \underbrace{\tfrac{1}{2}(1300)10^2 + \tfrac{1}{2}(1600)(6.6)^2}_{K_i} = 0$$

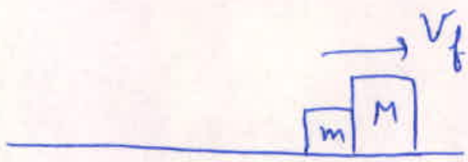
$$\Rightarrow x = \sqrt{0.303 \text{ m}^2}$$ {Put $k = 28{,}000$ N/m & $V_c = 8.12$ m/s}

$$\Rightarrow x = 0.55 \text{ m}$$

34.)



Case I.



Momentum Conservation →

$$m v_i = (m + M) v_f$$

$$\Rightarrow \frac{v_f}{v_i} = \frac{m}{m+M} \quad -①$$

Energy Conservation →

$$\frac{1}{2} m v_i^2 + 0 = \frac{1}{2} (m+M) v_f^2$$

$$\Rightarrow \frac{1}{\cancel{2}} m \cancel{v_i^2} = \frac{1}{\cancel{2}} (m+M) \frac{m^2}{(M+m)^2} \cancel{v_i^2}$$

(from eq ①)

$$\Rightarrow \frac{m}{m+M} = 1$$

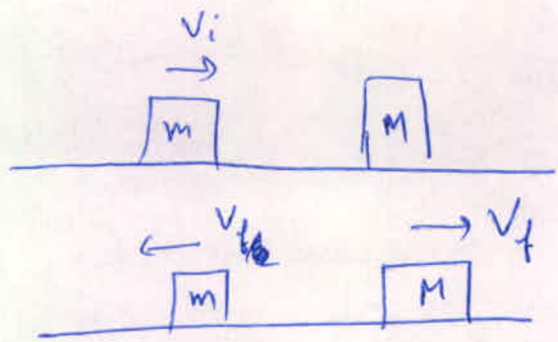
$$\Rightarrow m = m + M$$

$$\Rightarrow M = 0.$$

Not Possible.

∴ TAKE OTHER CASE

Case II



Momentum conservation →

$$m V_i = M V_f - m V_f$$

$$\Rightarrow V_f = \frac{m}{(M-m)} V_i \quad -(1)$$

Energy Conservation →

$$\frac{1}{2} m V_i^2 + 0 = \frac{1}{2}(m+M) V_f^2$$

$$\Rightarrow \frac{1}{2} m V_i^2 = \frac{1}{2}(m+M)\frac{m^2}{(M-m)^2} V_i^2 \quad \text{(from eq (2))}$$

$$\Rightarrow (M-m)^2 = (M+m)\, m$$

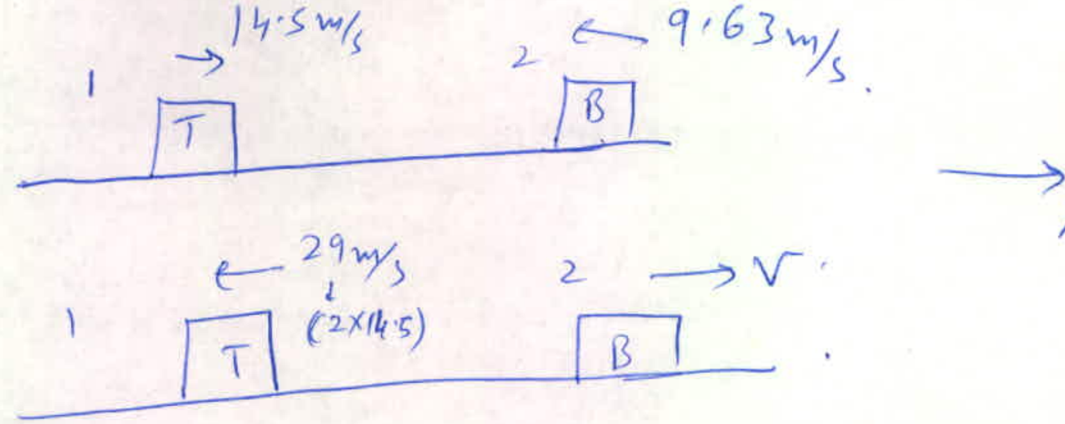
$$\Rightarrow M^2 + \cancel{m^2} - 2Mm = \cancel{m^2} + Mm$$

$$\Rightarrow M^{\cancel{2}} = 3\cancel{M}m$$

$$\Rightarrow M = 3m$$

$$\text{or} \quad \frac{M}{m} = 3$$

38.)



$\because$ its an elastic collision in 1-dimension,

We can directly write, (Formula from Chapter synopsis).

$$V_{2f} = V = \frac{2m_1}{m_1 + m_2} V_{1i} + \frac{m_2 - m_1}{m_1 + m_2} V_{2i}$$

Here, $m_1 = m_T$; $m_2 = m_B$; $V_{1i} = 14.5\,m/s$

$$V_{2i} = -9.63\,m/s$$

$$V_{1f} = -(2 \times 14.5)\ m/s = \frac{m_1 - m_2}{m_1 + m_2} V_{1i} + \frac{2m_2}{m_1 + m_2} V_{2i}$$

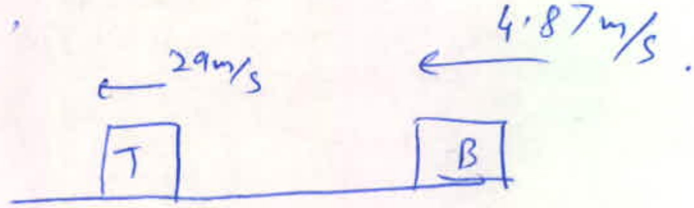
$$\Rightarrow -29 = \frac{59.1 - m_B}{59.1 + m_B}(14.5) + \frac{2m_B}{m_B + 59.1}(-9.63)$$

$$\Rightarrow m_B = 540\,g$$

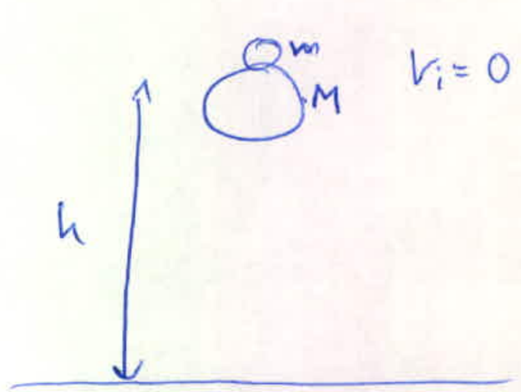
$$\therefore \quad V = \frac{2(59.1)}{59.1+540}(14.5) + \frac{(540-59.1)}{(540+59.1)}(-9.63)$$

$$\Rightarrow \quad V = -4.87 \text{ m/s}$$

$\therefore$ Finally,



64.)

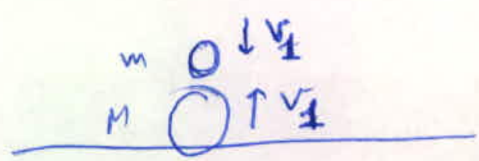


$$V_1^2 = 0 + 2gh$$

$$V_1 = \sqrt{2gh}$$

Large ball will collide first elastically with earth, and rebound with same velocity

$\therefore$



Now, the small ball will collide with large ball,

$\because$ its an elastic collision,

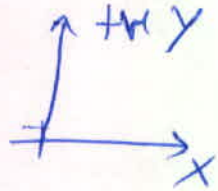
$$V_{1f} = \frac{m_1 - m_2}{m_1 + m_2} V_{1i} + \frac{2m_2 V_{2i}}{m_1 + m_2}$$

(See Chapter Synopsis)

$m_1 = m$, $m_2 = M$

$V_{1i} = -\sqrt{2gh}$

$V_{2i} = \sqrt{2gh}$



$$\therefore \quad V_{1f} = \frac{m - M}{m + M}\left(-\sqrt{2gh}\right) + \frac{2M}{m+M}\left(\sqrt{2gh}\right)$$

$$= \frac{\cancel{M}\left(m/M - 1\right)}{\cancel{M}\left(m/M + 1\right)}\left(-\sqrt{2gh}\right) + \frac{2\cancel{M}\left(\sqrt{2gh}\right)}{\cancel{M}\left(m/M + 1\right)}$$

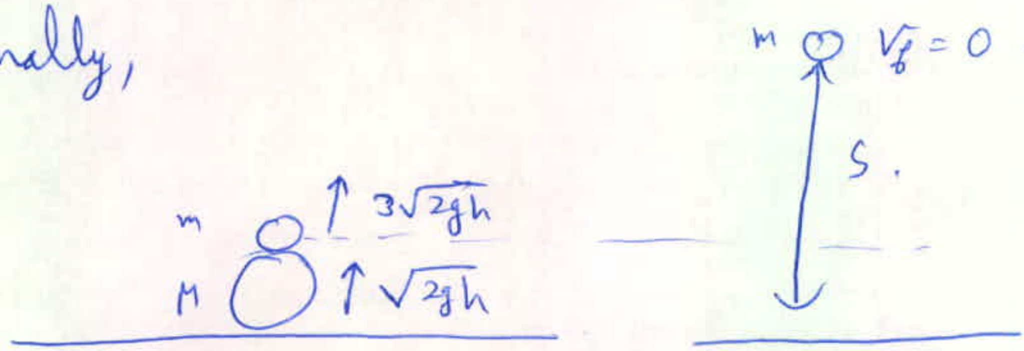
$\because \quad m << M$ or $\frac{m}{M} << 1$,

$$m/M - 1 \approx -1 \quad \& \quad m/M + 1 \approx 1$$

$$\therefore \quad V_{1f} = \frac{(-1)(-\sqrt{2gh})}{1} + \frac{2(\sqrt{2gh})}{1}$$

$$= 3\sqrt{2gh}$$

$\therefore$ Finally,



$\therefore$ Using $V^2 = u^2 + 2as$

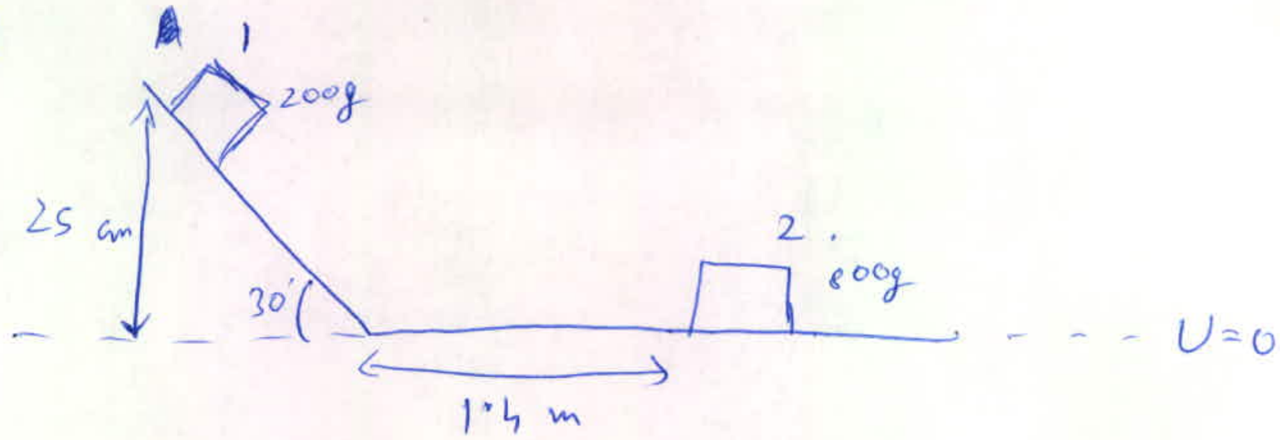
$u = 3\sqrt{2gh}$

$V = 0$

$\therefore \quad 0 = 9(2gh) - 2gS$

$\Rightarrow S = 9h$.

70.)



Applying energy conservation for block 1 just before it collides with 800g-block.

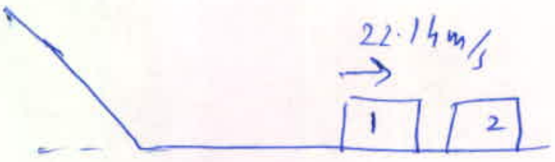
$$\Delta U + \Delta K = W_{nc} = 0.$$

$$\Rightarrow 0 - mg(25) + \frac{1}{2}mv^2 - 0 = 0$$

$$\Rightarrow v = \sqrt{2g25}$$

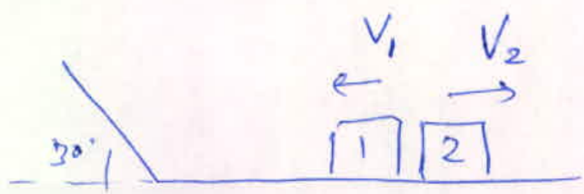
$$\Rightarrow v = 22.14 \text{ m/s}.$$

∴



After collision

$\longrightarrow$ +ve X



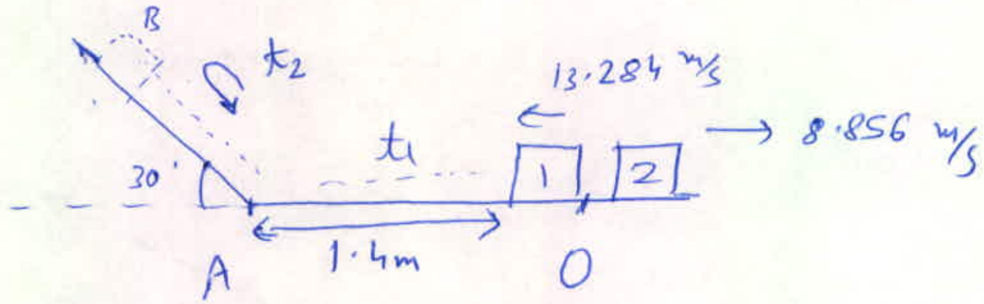
$$\therefore \; V_{1f} = (-V_1) = \frac{(200-800)}{(200+800)}(22 \cdot 14)$$

$$\Rightarrow V_1 = 13 \cdot 284 \text{ m/s}.$$

$$\& \; V_{2f} = V_2 = \frac{2 \times 200}{(800+200)} \times 22.14$$

$$\Rightarrow V_2 = 8 \cdot 856 \text{ m/s}$$

$\therefore$ After collision

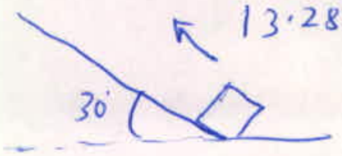


$t = 0$.

· time taken by block 1 (200g) to reach from point O to A.

$$t_1 = \frac{1 \cdot 4}{13 \cdot 284} \text{ s} = 0 \cdot 105 \text{ s}.$$

time taken by block to make the round-trip of going up and coming back down the incline be t_2



$\therefore \quad v = u + at$

$\Rightarrow -13.284\,(\sin 30^\circ) = (+13.284)\sin 30^\circ + (-g)\,t_2$

$\Rightarrow t_2 = \dfrac{2 \times 13.284 \sin 30^\circ}{g}$

Meanwhile, $\Rightarrow t_2 = 1.36\ s$.

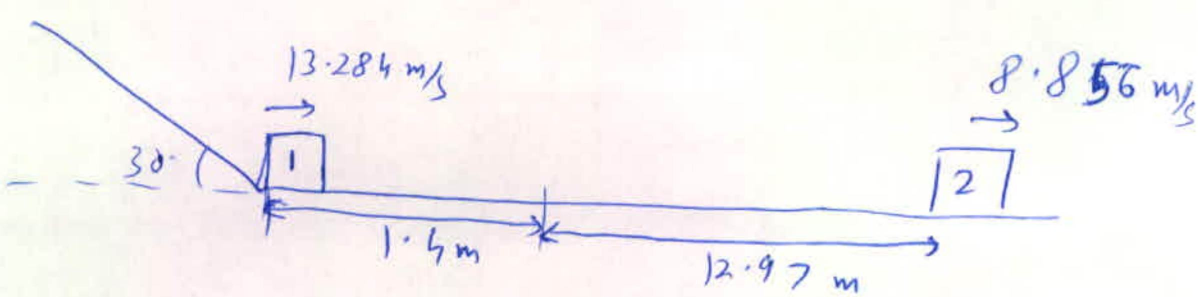
In time $(t_1 + t_2)$ Block 2 (800g) covered a distance of,

$d = (8.856)\ m/s \times (t_1 + t_2)$

$= 8.856\,(1.36 + 0.105)$

$= 12.97\ m$.

$\therefore$ Now



$\therefore$ distance between the blocks $= 1.4 + 12.97$

$= 14.37 \text{ m}$

$|V_{rel}| = (13.284 - 8.856) \text{ m/s}$

$= 4.428 \text{ m/s}$

$\therefore$ $\left.\begin{array}{l}\text{Time, when they collide} \\ \text{again } (t_3)\end{array}\right\} = \dfrac{14.37}{4.428} \text{ s}$

$= 3.24 \text{ s}$

$\therefore$ Total time taken

$= t_1 + t_2 + t_3$

$= (1.36 + 0.105 + 3.24) \text{ s}$

$= 4.705 \text{ s}$