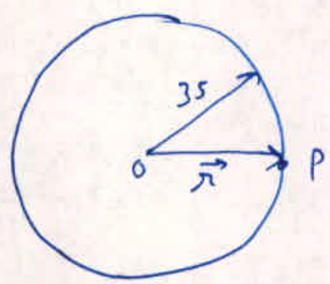
HW #7

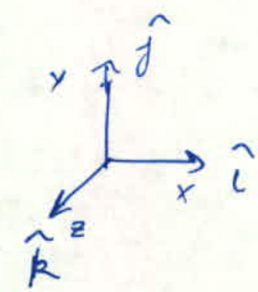
PHYS 4A

WINTER '15

Chapter - 13.

8.) a)



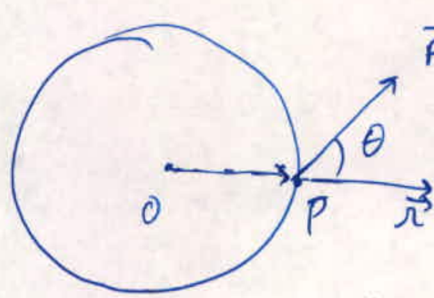


$\therefore \hat{\tau} = \hat{k}$

& $\vec{\tau} = \vec{r} \times \vec{F}$

$\hat{F}$ should be in x-y Plane

& $\vec{r}$ is in $\hat{i}$ direction



$$|\vec{\tau}| = |\vec{r}|\,|\vec{F}| \sin\theta$$

$$\Rightarrow |\vec{F}| = \frac{|\vec{\tau}|}{|\vec{r}| \sin\theta}$$

for $|\vec{F}|$ to be minimum, $\sin\theta$ should be maximum. $\therefore \theta = 90^\circ \Rightarrow \sin\theta = 1$

$$\therefore |\vec{F}| = \frac{|\vec{\tau}|}{|\vec{r}| \times 1} = 3.43 \text{ N}.$$

$$\therefore \vec{F} = 3.43 \hat{j} \text{ N}.$$

b) Now, given $\hat{\tau} = \hat{j}$ or $-\hat{j}$

$\therefore$ $\hat{F}$ should be $\hat{k}$ or $-\hat{k}$

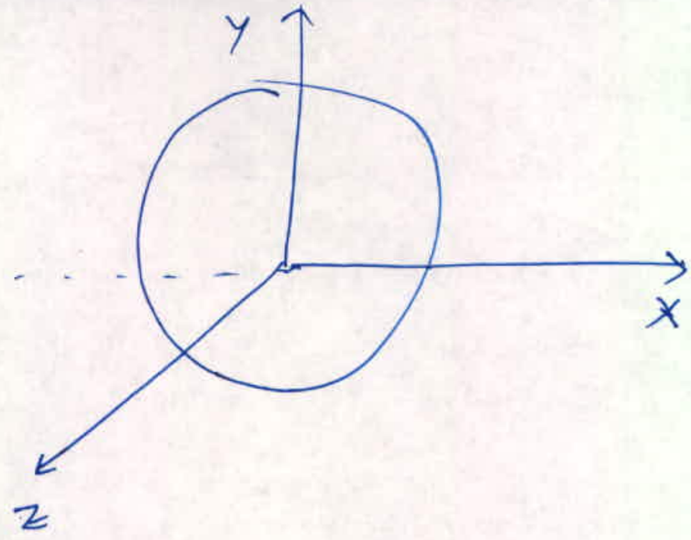
{ Using same argument as in part a).
$|\vec{F}|$ is minimum when angle between
$\vec{r}$ & $\vec{F}$ is 90° }

$$\therefore |\vec{F}| = \frac{|\vec{\tau}|}{|\vec{r}| \times 1} = 3.43 \text{N}$$

$$\vec{F} = 3.43 \text{ N } \underbrace{\hat{k} \ (\text{or} -\hat{k})}$$

depending on whether
$\vec{\tau}$ is in $\hat{j}$ or $-\hat{j}$.

10.)



$|\vec{r}| = 1\,m$

a) $\vec{\tau} = -5\hat{k}$

& $\vec{F} = 5\hat{j}$

$\therefore \quad \vec{\tau} = \vec{r} \times \vec{F}$

$\Rightarrow -5\hat{k} = \vec{r} \times 5\hat{j}$

we know that $-\hat{i} \times \hat{j} = -\hat{k}$

& $|\vec{r}| = 1$

$\therefore \quad \vec{r} = 1(-\hat{i})$

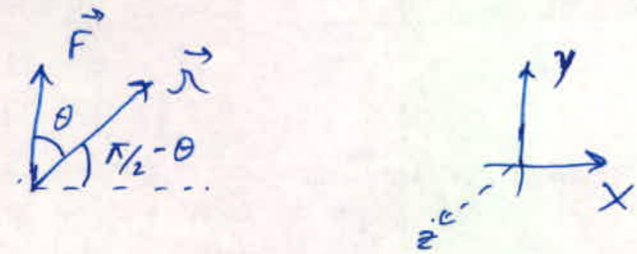
b) $\vec{\tau} = 3.4\,\hat{k}$ & $\vec{F} = 5\hat{j}$

$|\vec{\tau}| = |\vec{r}|\,|\vec{F}|\sin\theta$

$\Rightarrow \sin\theta = \dfrac{3.4}{5}$

$\Rightarrow \theta = 42.84^\circ$

i.e. angle between $\vec{r}$ & $\vec{F}$ is 42.84°



$$\therefore \vec{r} = |\vec{r}| \left[\cos\left(\tfrac{\pi}{2}-\theta\right) \hat{i} + \sin\left(\tfrac{\pi}{2}-\theta\right) \hat{j}\right]$$

$$= 1 \left[\sin\theta \, \hat{i} + \cos\theta \, \hat{j}\right]$$

$$= 0.68 \, \hat{i} + 0.73 \, \hat{j}$$

This will give $\vec{\tau} = -3.4 \hat{k}$. Hence, $\vec{r} = 0.68\hat{i} - 0.73\hat{j}$.

c) $\vec{\tau} = -3.4 \, \hat{i}$

Now, $\vec{r}$ should be in Y-Z plane as $\vec{\tau}$ is in $-X$ direction.

$$\vec{r} = |\vec{r}| \left[\cos\theta \, \hat{j} + \sin\theta \, \hat{k}\right]$$

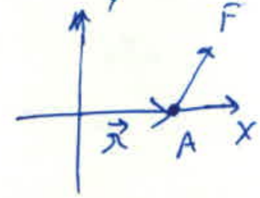
$$= 1 \left[0.73 \, \hat{j} + 0.68 \, \hat{k}\right]$$

Check: $\vec{r} \times \vec{F} = -3.4 \, \hat{i}$

$$\therefore \vec{r} = 0.73 \, \hat{j} + 0.68 \, \hat{k}$$

12.) $\vec{F} = 1.3\hat{i} + 2.7\hat{j}$

a) $x = 3$ m & $y = 0$ about origin



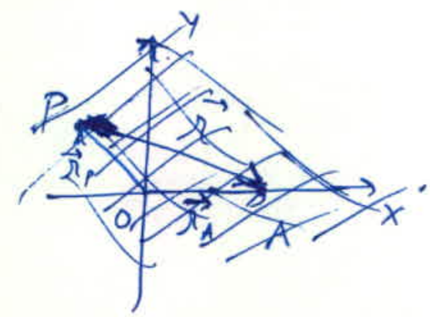
$\therefore \vec{r} = 3\hat{i}$

$\therefore \vec{\tau} = \vec{r} \times \vec{F}$

$= 3\hat{i} \times (1.3\hat{i} + 2.7\hat{j})$

$= 8.1\,\hat{k}$ Nm

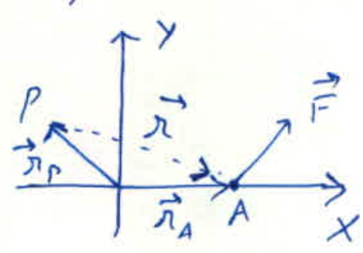
b) $x = -1.3$ m, $y = 2.4$ m $\rightarrow P$



$\therefore \vec{r}_P + \vec{r} = \vec{r}_A$

$\Rightarrow (-1.3\hat{i} + 2.4\hat{j}) + \vec{r} = 3\hat{i}$

$\Rightarrow \vec{r} = 4.3\hat{i} - 2.4\hat{j}$



$\therefore \vec{\tau} = \vec{r} \times \vec{F}$

$= (4.3\hat{i} - 2.4\hat{j}) \times (1.3\hat{i} + 2.7\hat{j})$

$= 14.73\,\hat{k}$ Nm.

20.) a) $$I = \sum_{i=1}^{4} I_i$$

$$= 4 \times \left[M \left(\frac{16}{100} \right)^2 \right]$$

$$= 4 \times \frac{120}{1000} \times \frac{256}{10^4}$$

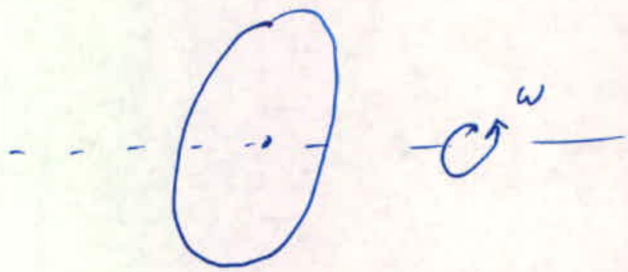
$$= 0.0123 \text{ kg m}^2.$$

b) $$|\vec{L}| = I |\vec{\omega}|$$

$$= 0.0123 \times (12 \times 2\pi) \text{ rad/s}$$

$$= 0.9274 \text{ kg m}^2 \text{ rad/s}.$$

22.)



$$\omega = 170 \text{ rpm} = 170 \times \frac{2\pi}{60} \text{ rad/s}$$

$$= \frac{17\pi}{3} \text{ rad/s}$$

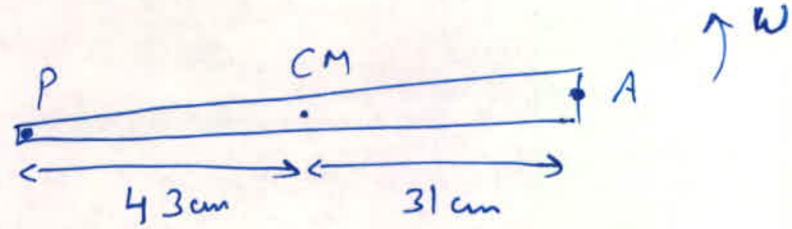
$$|\vec{L}| = I\omega$$

$$= MR^2 \omega$$

$$= \frac{640}{1000} \times \left(\frac{45}{100} \right)^2 \times \frac{17\pi}{3}$$

$$= 2.31 \text{ kg m}^2 \text{ rad/s}$$

28.) a)



$$L_{(\text{about } P)} = I_P \, \omega_P$$

$$I_P = I_{cm} + M\left(\frac{43}{100}\right)^2$$

$$\cancel{= \frac{M}{12}\left(\frac{43+31}{100}\right)^2 + M\left(\frac{43}{100}\right)^2}$$

$$= 0.048 + 0.88\,(0.43)^2$$

$$= 0.211 \ \text{kg m}^2$$

$$\omega_P = \frac{v_A}{r_A} = \frac{50}{\left(\frac{43+31}{100}\right)} \ \text{rad/s} = 67.57 \ \text{rad/s}$$

$$\therefore \ L_P = I_P \, \omega_P$$

$$= (0.211)(67.57)$$

$$= 14.24 \ \text{kg m}^2 \ \text{rad/s}.$$

b) $\frac{d\vec{L}}{dt} = \vec{\tau}$

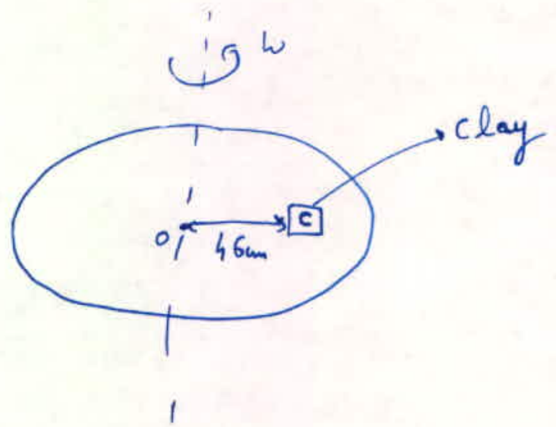
$\therefore \int dL = \int dt\, \tau$

$\because \tau$ is constant,

$\therefore \Delta L = \tau t$

$\Rightarrow \tau = \frac{\Delta L}{t} = \frac{14.24}{0.25} = 56.96\ Nm$

30.)



$\because \vec{\tau}_{ext} = 0$ on the whole system,

$\therefore$ Angular Momentum is conserved.

i.e. $L_i = L_f$

Ang. Momentum of Potter Wheel

$\Rightarrow I_{pw}\, \omega_{pw_i} = I_{pw}\, \omega_{pw_f} + I_c\, \omega_{c_f}$

Angular momentum of Clay.

$\because$ Clay sticks to the wheel

$\therefore \omega_{c_f} = \omega_{pw_f}$

$$\Rightarrow \quad 6.4 \times 19 \text{ rpm} = 6.4 \times \omega_{pw_f} + 2.7 \times (0.46)^2 \omega_{pw_f}$$

$$\Rightarrow \quad \omega_{pw_f} = \frac{6.4 \times 19}{\left[6.4 + 2.7(0.46)^2\right]} \text{ rpm}$$

$$= 17.44 \text{ rpm}$$

42.)

$$\Omega = 0.42 \text{ rad/s}$$

$$\tau = 0.31 \text{ Nm}$$

Assuming $\omega >> \Omega$. as it is valid only in that case.

$$\therefore \quad \Omega = \tau / L$$

$$\Rightarrow L = \tau / \Omega = \frac{0.31}{0.42} = 0.738 .$$

A $L = I\omega$

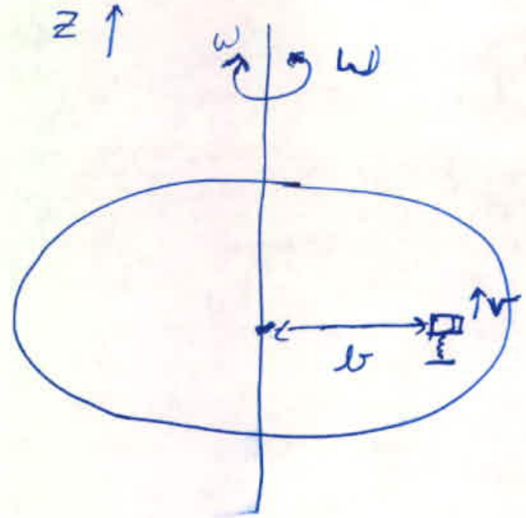
$$\Rightarrow \omega = \frac{0.738}{I} = \frac{0.738}{6.8 \times 10^{-3}} \text{ rad/s}$$

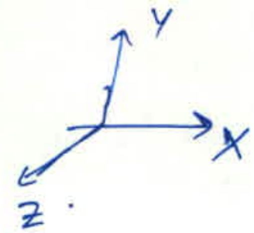
$$\Rightarrow \omega = 108.54 \text{ rad/s}$$

<u>Check:</u> $\omega >> \Omega$

$$108.54 >> 0.42$$

56.)





a) Conserving angular momentum between the points →

when the spring is compressed &

when the mass leaves with speed v.

$\therefore \quad L_i = L_f$

$L_i = 0 \rightarrow$ everything is at rest.

$\therefore \quad L_f = 0$

$\Rightarrow \quad L_{Mass} + L_{Table} = 0$

$\Rightarrow \quad m v b (\hat{k}) + I \omega (-\hat{k}) = 0$

$\Rightarrow \quad m v b = I \omega$

has to be in $-\hat{k}$ so that $L_f = 0$.

$$\Rightarrow \quad \omega = \frac{m v b}{I} \quad \text{—①}$$

Conserving Energy,

$$\Delta K + \Delta U = W_{nc} = 0.$$

$$\Rightarrow K_f - \cancel{K_i}^{\,0} + \cancel{U_f}^{\,0} - U_i = 0$$

$$\Rightarrow \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2 = U_i = \frac{1}{2} k (\Delta x)^2$$

$$\Rightarrow \cancel{\frac{1}{2}} m v^2 + \cancel{\frac{1}{2}} \cancel{I} \frac{(m v b)^2}{I^{\cancel{2}}} = \cancel{\frac{1}{2}} k (\Delta x)^2$$

(from eq ①)

$$\Rightarrow v^2 \left[m + \frac{m^2 b^2}{I} \right] = k (\Delta x)^2$$

$$\Rightarrow v = \left[\frac{k \Delta x^2 I}{m (I + m b^2)} \right]^{1/2}$$

b) $$\omega = \frac{m v b}{I} \quad \text{(from eq ①)}$$

$$= \frac{m b}{I} \left[\frac{k \Delta x^2 I}{m (I + m b^2)} \right]^{1/2}$$