Physics 4A
Lecture 4: Jan. 15, 2015

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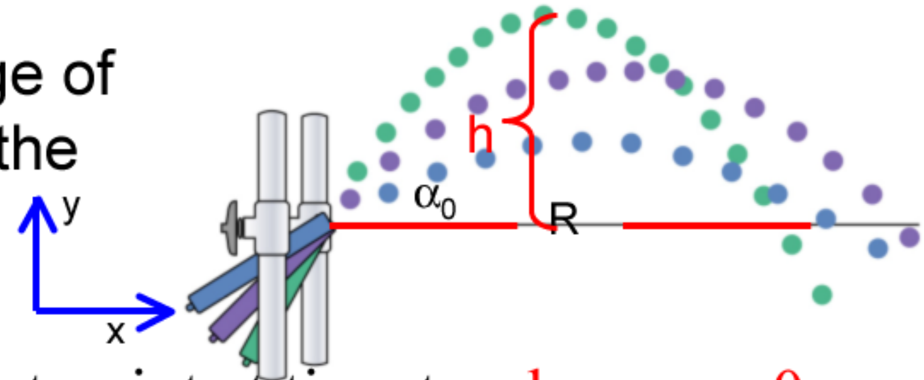
UCSD Physics

Height & Range of Projectiles

Max. height & the range of projectile depends on the firing angle α_0



$$\text{Projectile at its highest point at time } t_1 \text{ when } v_y = 0$$

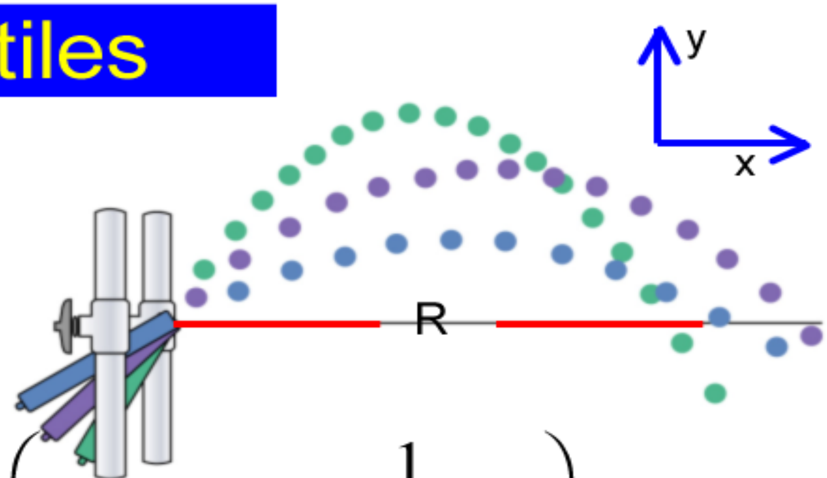
$$\Rightarrow v_y = v_0 \sin\alpha_0 - g t_1 = 0 \Rightarrow t_1 = \frac{v_0 \sin\alpha_0}{g}$$

$$\text{at this time, } y = h = v_0 \sin\alpha_0 \frac{v_0 \sin\alpha_0}{g} - \frac{1}{2} g \left(\frac{v_0 \sin\alpha_0}{g} \right)^2$$

$$\Rightarrow \boxed{h = \frac{v_0^2 \sin^2\alpha_0}{2g}}; \quad \text{largest at } \alpha_0 = 90^0 \text{ (vertical launch)}$$

Range of Projectiles

R is the projectile's x location at some $t=t_2$ when $y=0$



$$0=(v_0 \sin\alpha_0)\, t_2 - \frac{1}{2} g t_2^2 = t_2\left(v_0 \sin\alpha_0 - \frac{1}{2} g t_2\right) = 0$$

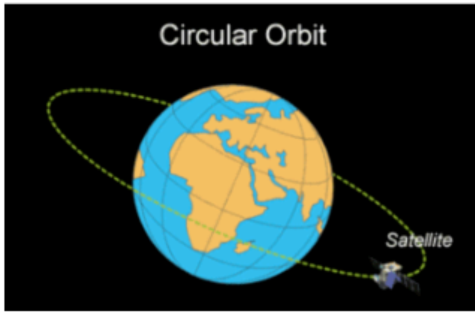
Two solutions for t_2 : $\boxed{t_2 = 0}$ & $\boxed{t_2 = \frac{2 v_0 \sin\alpha_0}{g}}$

$$\text{Range } R = v_0 \cos\alpha_0 . \frac{2 v_0 \sin\alpha_0}{g} = \boxed{\frac{v_0^2 \sin 2\alpha_0}{g} = R}$$

$R = R_{max}$ when $\alpha_0 = 45^o$

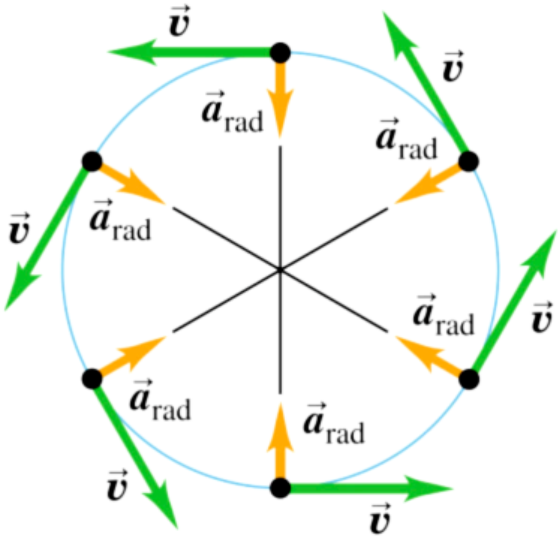
(Using trig. identity: $2\sin\theta\cos\theta = \sin 2\theta$)

Uniform Circular Motion



Very different from projectile motion where accel. was const. and **always** in 1 direction

In uniform circular motion the speed of object is constant but velocity is always changing

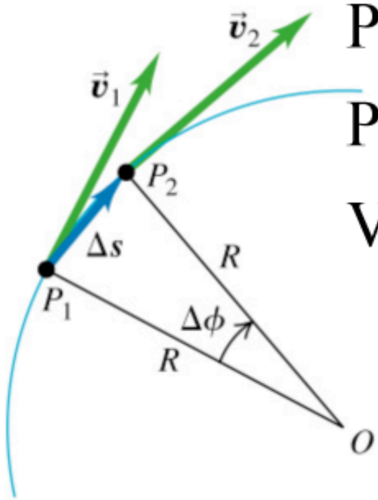


No component of accel. parallel (or tangent) to path so no change in speed

$$\vec{a} = \vec{a}_\perp = \vec{a}_r$$

Component of accel $\perp$ to path causes direction of velocity to change

Radial Component of Acceleration



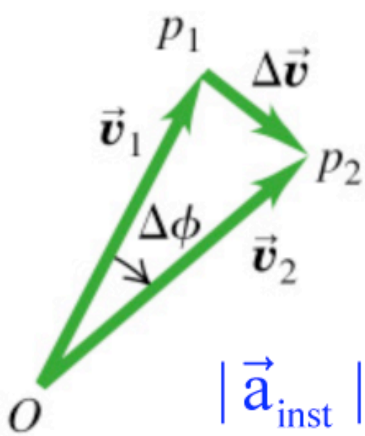
Particle moving in circle of radius R around O

Particle moves from $P_1 \rightarrow P_2$ in time Δt

Velocity changes from $\vec{v}_1 \rightarrow \vec{v}_2$ in time Δt

Triangles OP_1P_2 & Op_1p_2 are *similar* $\Rightarrow$

Ratio of corresponding sides are equal



$$\Rightarrow \frac{\Delta\vec{v}}{|\vec{v}_1|} = \frac{\Delta s}{R} \Rightarrow \Delta\vec{v} = \frac{\Delta s}{R}|\vec{v}_1|$$

$$\Rightarrow \quad |\vec{a}_{av}| = \frac{\Delta\vec{v}}{\Delta t} = \frac{|\vec{v}_1|}{R}\frac{\Delta s}{\Delta t}$$

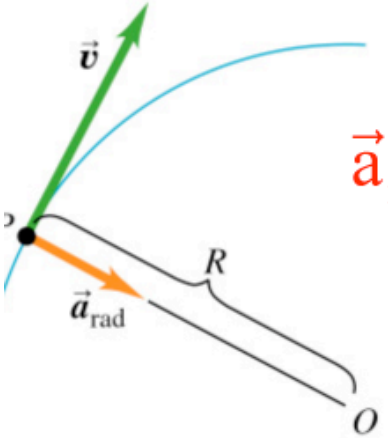
$$|\vec{a}_{inst}| = \lim_{\Delta t \rightarrow 0} \frac{|\vec{v}_1|}{R}\frac{\Delta s}{\Delta t} = \frac{|\vec{v}_1|}{R} \lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t} = \frac{|\vec{v}_1||\vec{v}_1|}{R} = \boxed{\frac{v^2}{R}}$$

Radial Component of Acceleration

$|\vec{a}_{inst}| = |\vec{a}_{rad}| = \boxed{\dfrac{v^2}{R}}$ also called Centripetal Acceleration

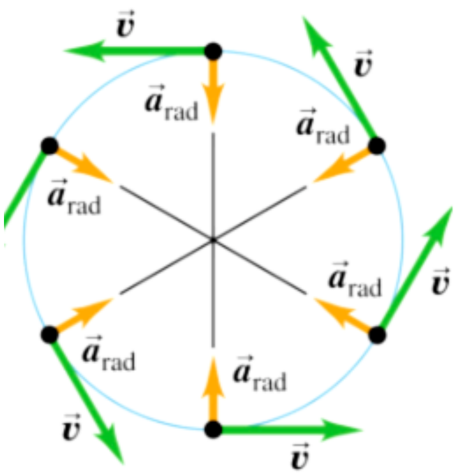
Centripetal = "seeking the center"

$\vec{a}_{rad}$ is $\perp$ to $\vec{v}$ and directed radially *inwards*



Period of Motion :T

In time T, particle makes one full trip around circle. So its speed $\boxed{|\vec{v}| = \dfrac{2\pi R}{T}}$



But since $|\vec{a}_{rad}| = \dfrac{v^2}{R} \Rightarrow \boxed{|\vec{a}_{rad}| = \dfrac{4\pi^2 R}{T^2}}$

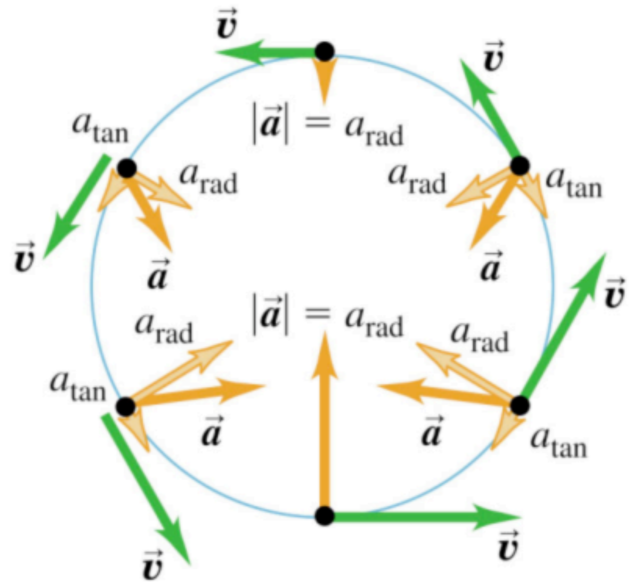
Pop Quiz:

- A satellite is moving in a circular orbit of radius 9 x 10^3 Km around the Earth with a speed of 20 m/s. Suddenly it is kicked into an orbit of 18 x 10^3 Km with a speed of 40 m/s

Its acceleration towards Earth

- (A) Does not change
- (B) Increases by a factor of 2
- (C)Decreases by a factor of 2
- (D) Decreases by a factor of √2

Non-Uniform Circular Motion



Here speed changes along the circular path

$$a_{||} = a_{tangent} = \frac{d|\vec{v}|}{dt} \neq 0 \ \ \& \ a_r = \frac{v^2}{R}$$

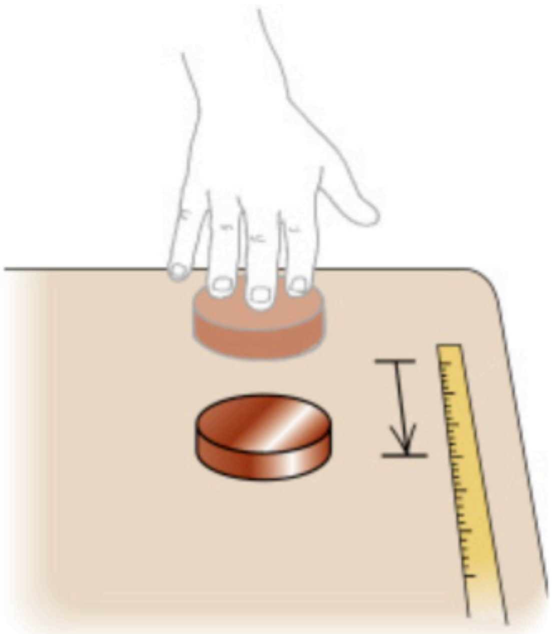
a_{rad} is largest when speed is largest, smallest when speed is smallest.

a_{tan} is || to $\vec{v}$ (going downhill) and anti-|| $\vec{v}$ when object going uphill !

- Kinematics:
  - *Language* for describing motion
    - displacement, velocity, acceleration
- Dynamics:
  - Cause of change in motion
    - New Concepts : Force and Mass
    - Newton’s Three Laws of Motion
      - Obtained from first principles $\Rightarrow$ Fundamental
      - Based on experimental observations of motion, NOT product of some math derivation

Intuition About Motion May Misguide You!

Motion of a hockey puck on a horizontal table.
Puck stops shortly after you stop pushing

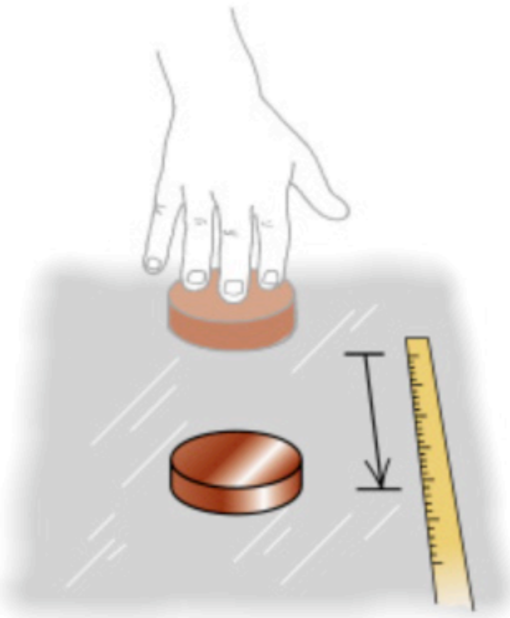


(a) Table:
puck stops short

Many, like Aristotle,
would conclude :
Bodies in motion
naturally come to rest
&
A force must act on puck
to sustain motion

Idealization of A Motion

Examine motion of a hockey puck on a waxed floor under a *similar* push (force)



(b) Waxed floor: puck slides farther

Puck now travels much farther before it stops

What's different is that the surface has smaller friction← the interaction between the belly of puck & floor retards puck's motion

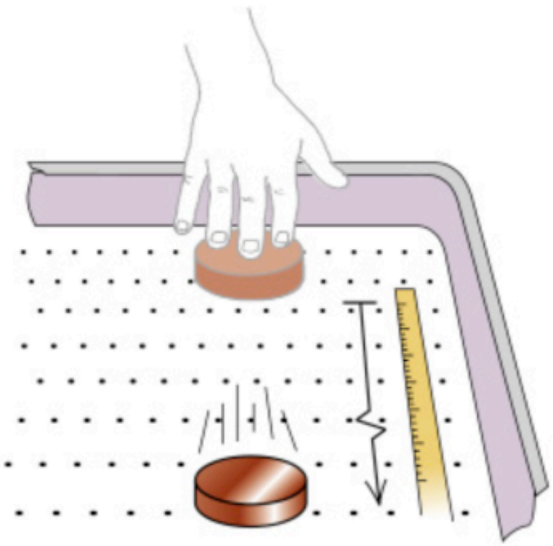
Intuition Not A Reliable Guide !

Examine motion of the hockey puck under a similar push (force) on an air-hockey table

Puck now travels even farther before it stops

What's different is that the surface has even smaller friction

In absence of friction, puck would never slow down and we wont need a force to keep it moving



(c) Air-hockey table: puck slides even farther

Newton's First Law Of Motion

- Actually first noticed by Galileo in his experiments with smooth surfaces (technology!)
- Later generalized by Newton:

When there is no net force acting on an object, the object maintains its motion with a constant velocity (zero acceleration)

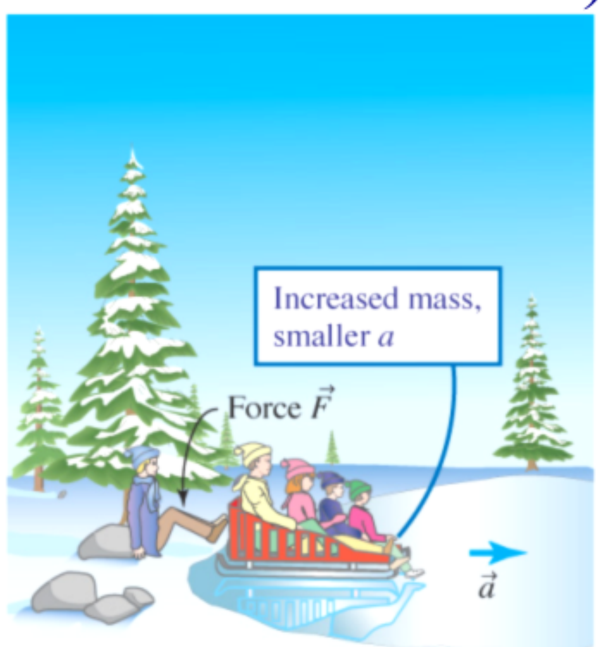
Conversely:
When an object moves with constant velocity, the net force acting on it must be zero

Tendency of an object to maintain its state of rest or uniform motion is called *inertia*

Newton's 2nd Law Of Motion

$$\vec{F} \propto \vec{a} \quad ; \quad \vec{F} = m\vec{a} \text{ or } \vec{a} = \frac{\vec{F}}{m}$$

m = inertial mass= measure of an object's resistence to change in its motion (The "Inertia" of 1st law)



Mass Of An Object

Ratio $\frac{\left|\sum \vec{F}\right|}{|\vec{a}|}$ is property of the object & is constant

regardless of the magnitude of the applied force

The larger the mass of an object, harder it is

to change its velocity. e.g.: Truck Vs Baseball

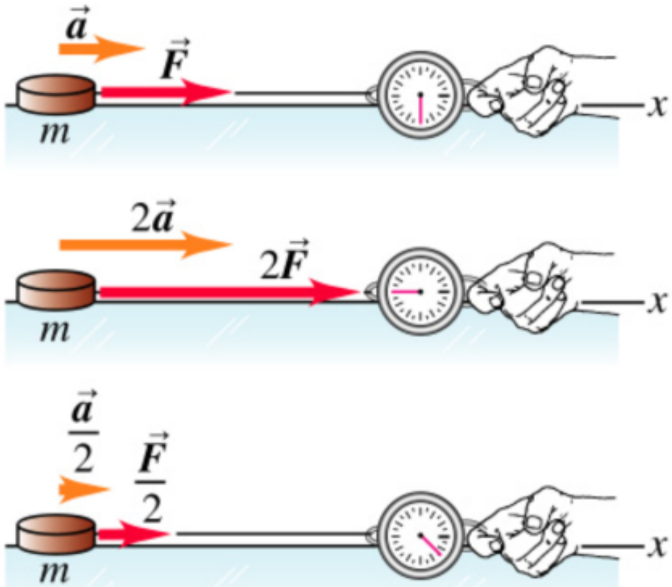
Definition of inertial mass: $m = \frac{\left|\sum \vec{F}\right|}{|\vec{a}|}$; SI unit = kg

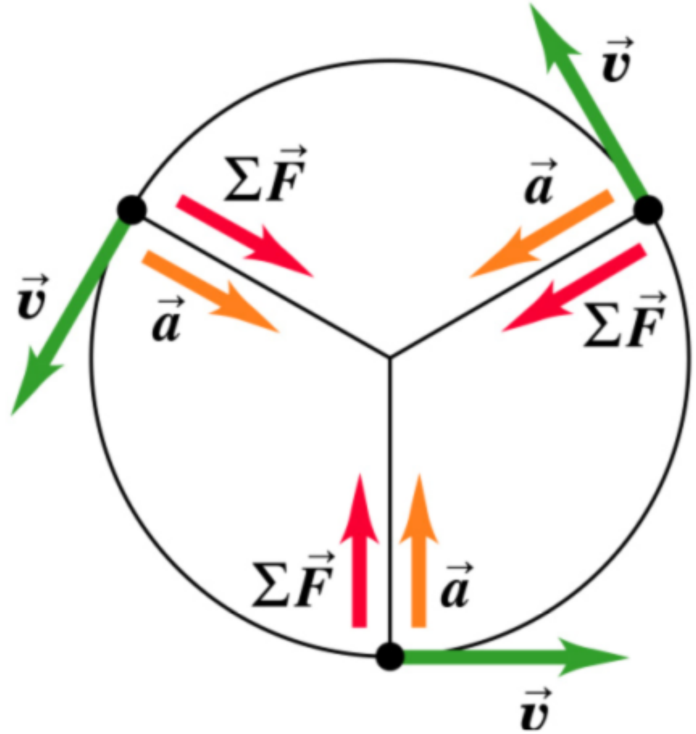
The definition tells us how to measure mass

No matter where you measure it, Earth, Mars or Pluto, mass of an object is always the same !

Newton's 2nd Law Of Motion

$$\vec{F} \propto \vec{a}$$





At all points in circular motion,

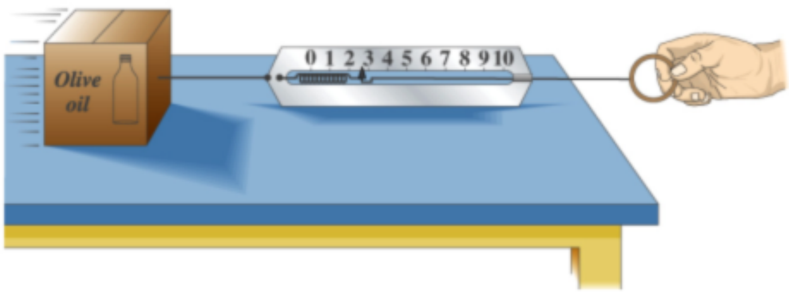
$\vec{a}$ & $\sum\vec{F}$ are in same direction:

towards the center

Measuring Mass

- To measure mass, "you" have to "mess" with object

measure change
in its motion
due to applied force

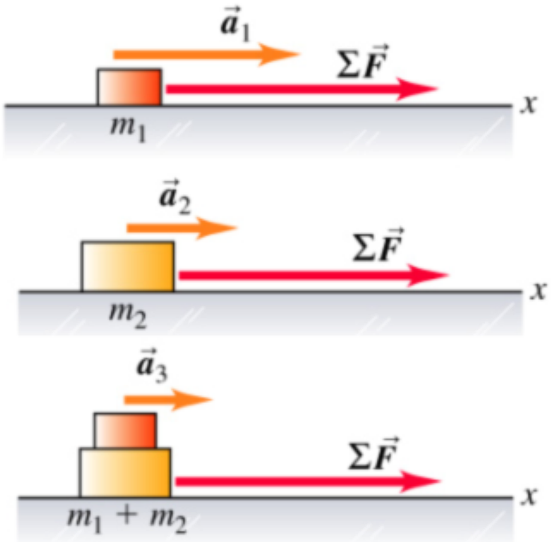


- Masses add like scalars

$$\vec{a}_1 = \sum \vec{F} / m_1$$

$$\vec{a}_2 = \sum \vec{F} / m_2$$

$$\vec{a}_3 = \sum \vec{F} / (m_1 + m_2)$$



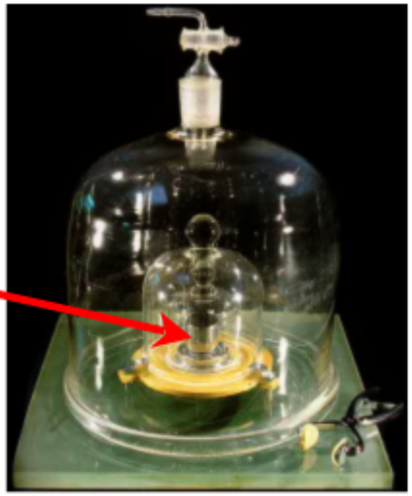
More on Force

$\boxed{\vec{F} = m\vec{a}}$ is NOT the DEFINITION of Force!

Equation shows the reaction of the object to a force

SI Unit of force is the NEWTON

1 Newton is the amount of net force that gives an acceleration $a = 1\ m/s^2$ to an object of mass $m = 1.0$ kg



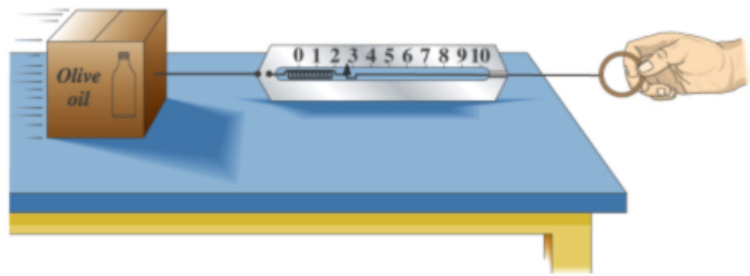
$1\ N = 1 kg .m/s^2$

In CGS units: 1 dyne = 1 g.cm/s^2 = 10^{-5} Newton

In British units: 1 Pound=1 slug.ft/s^2 = 4.45N

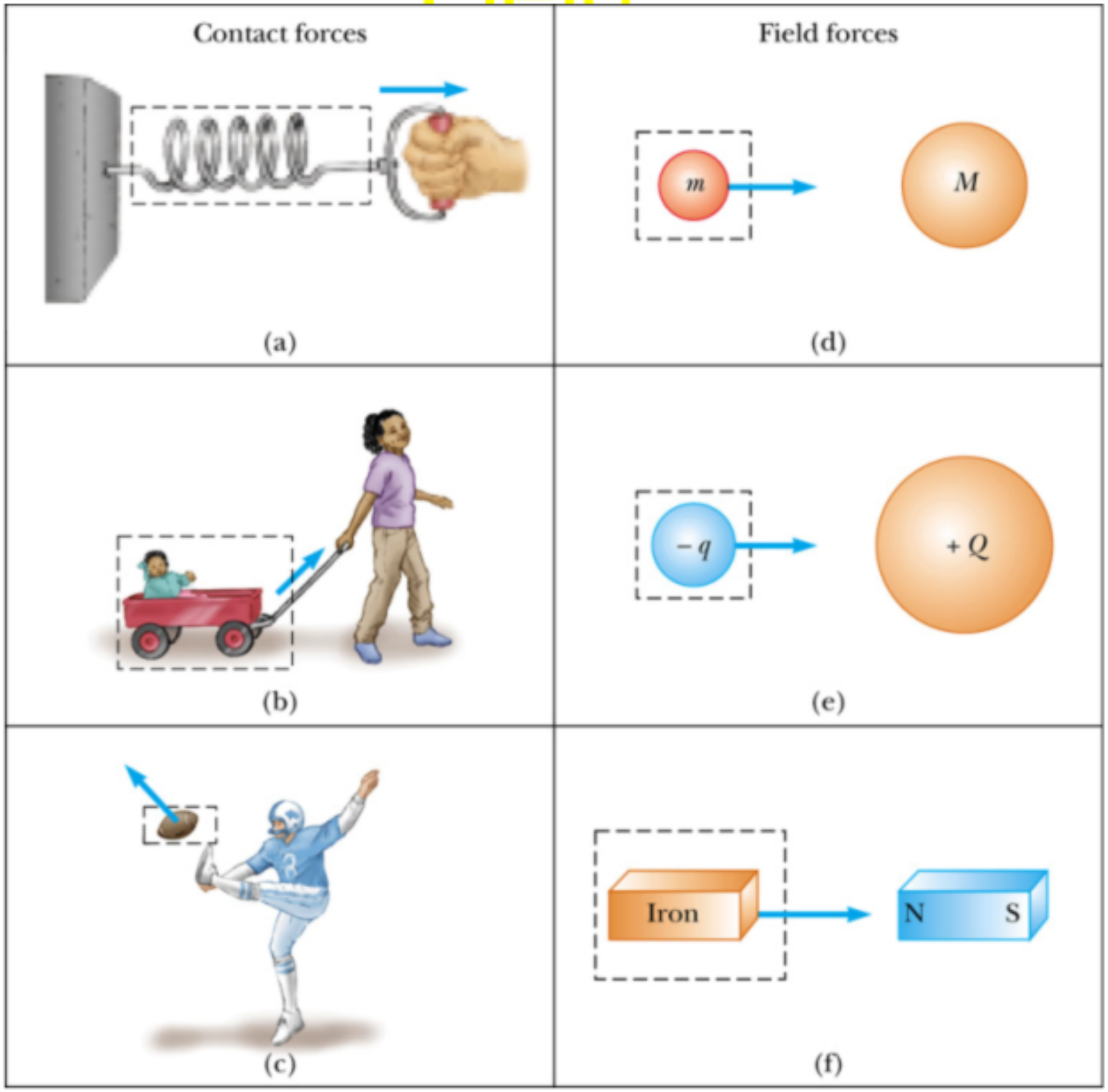
The Force Vector

- Force is a *push or pull* on some object
- Gives quantitative description of interaction between an object(s) & its environment
  - i.e: pulling a box



- Force is a vector:
  - has direction and magnitude
  - SI unit of magnitude is the Newton (N)
  - Force of gravitational attraction that earth exerts on your body is your weight

Two Kinds Of Forces: Contact & Field



Typical Force Magnitude

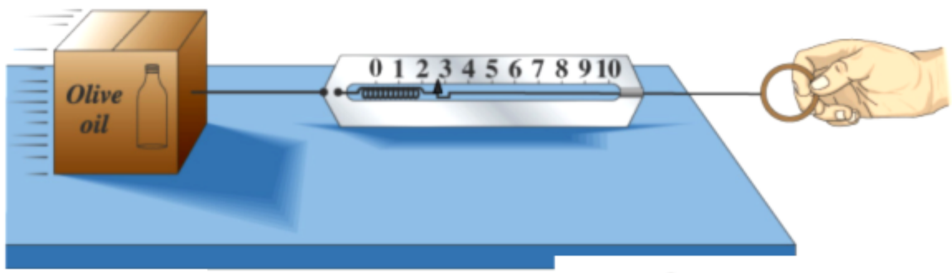
Sun's gravitational force on the earth	3.5×10^{22} N
Thrust of a space shuttle during launch	3.1×10^{7} N
Weight of a large blue whale	1.9×10^{6} N
Maximum pulling force of a locomotive	8.9×10^{5} N
Weight of a 250-lb linebacker	1.1×10^{3} N
Weight of a medium apple	1 N
Weight of smallest insect eggs	2×10^{-6} N
Electric attraction between the proton and the electron in a hydrogen atom	8.2×10^{-8} N
Weight of a very small bacterium	1×10^{-18} N
Weight of a hydrogen atom	1.6×10^{-26} N
Weight of an electron	8.9×10^{-30} N
Gravitational attraction between the proton and the electron in a hydrogen atom	3.6×10^{-47} N

Instrument For Measuring Forces

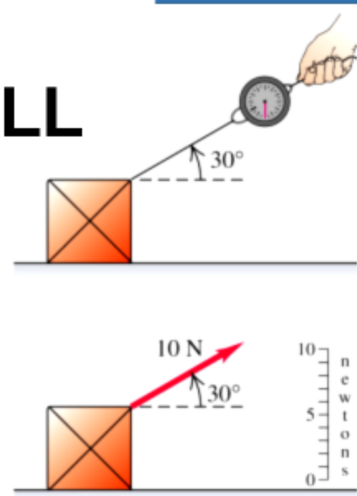
Spring balance is coil spring attached to a pointer

When force applied → spring stretches some.

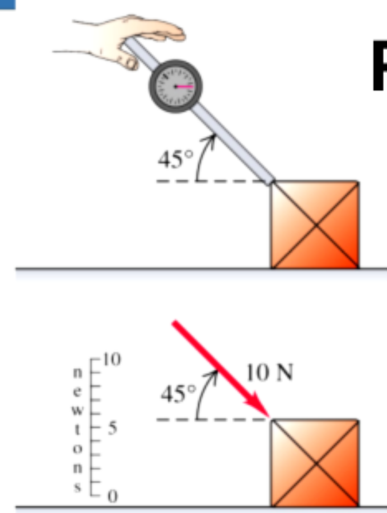
Amount of stretch directly proportional to force



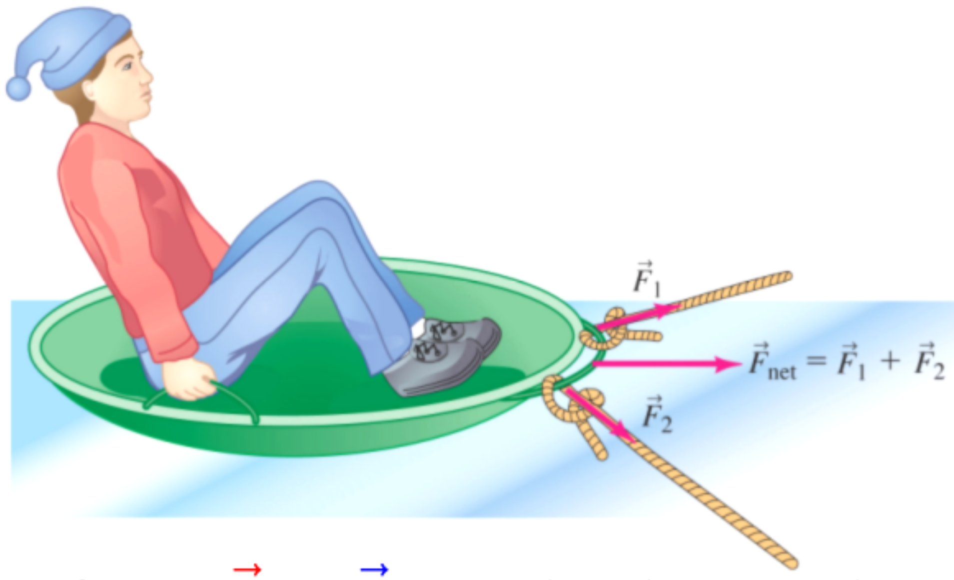
PULL



PUSH



Superposition of Forces



When two forces $\vec{F}_1$ & $\vec{F}_2$ act simultaneously on an object, experiments show that the net effect on object's motion is same as effect of *single* force: $\vec{F}_{net} = \vec{R} = \vec{F}_1 + \vec{F}_2$

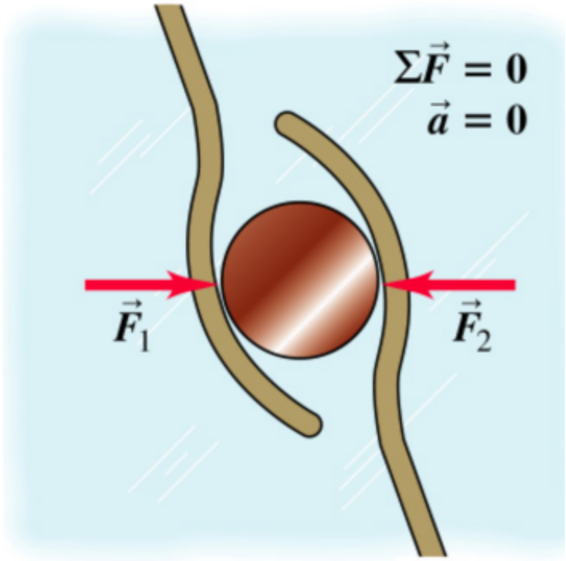
Generally true for any number n of forces: $\vec{F}_{net} = \vec{R} = \sum_n \vec{F}_n$

Equilibrium Condition For Motion

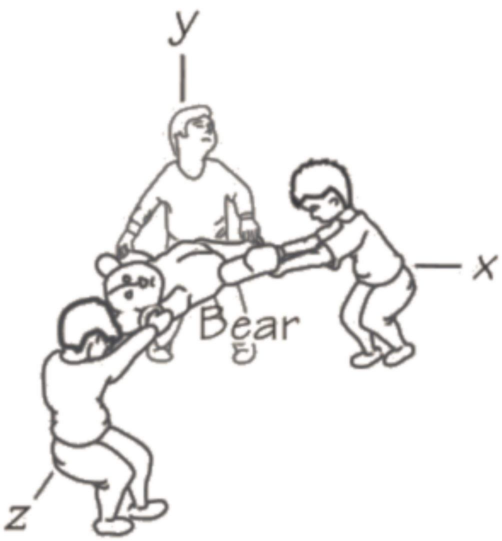
When an object is acted on by no force or by several forces such that net force=0, the object is said to be in equilibrium. In this state body is either at rest or moving in a straight line with constant velocity

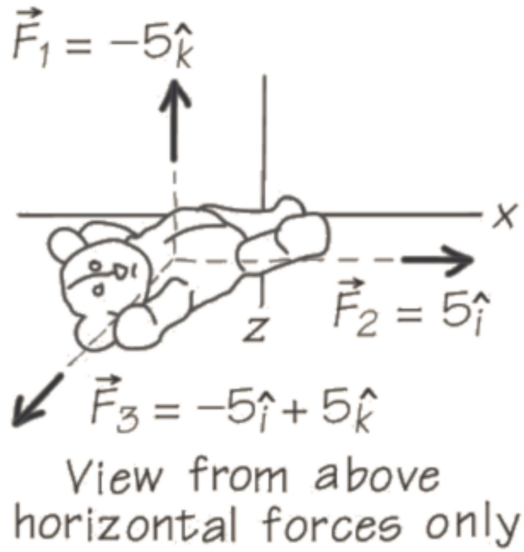
$$\sum \vec{F} = 0 \qquad \text{(body in equilibrium)}$$

$$\Rightarrow \sum \vec{F}_x = 0 = \sum \vec{F}_y$$



Net Force $\vec{F}_{net} = \vec{R} = \sum_{n} \vec{F}_n$





Net force is the vector sum of the three forces tugging at the bear in the horizontal plane

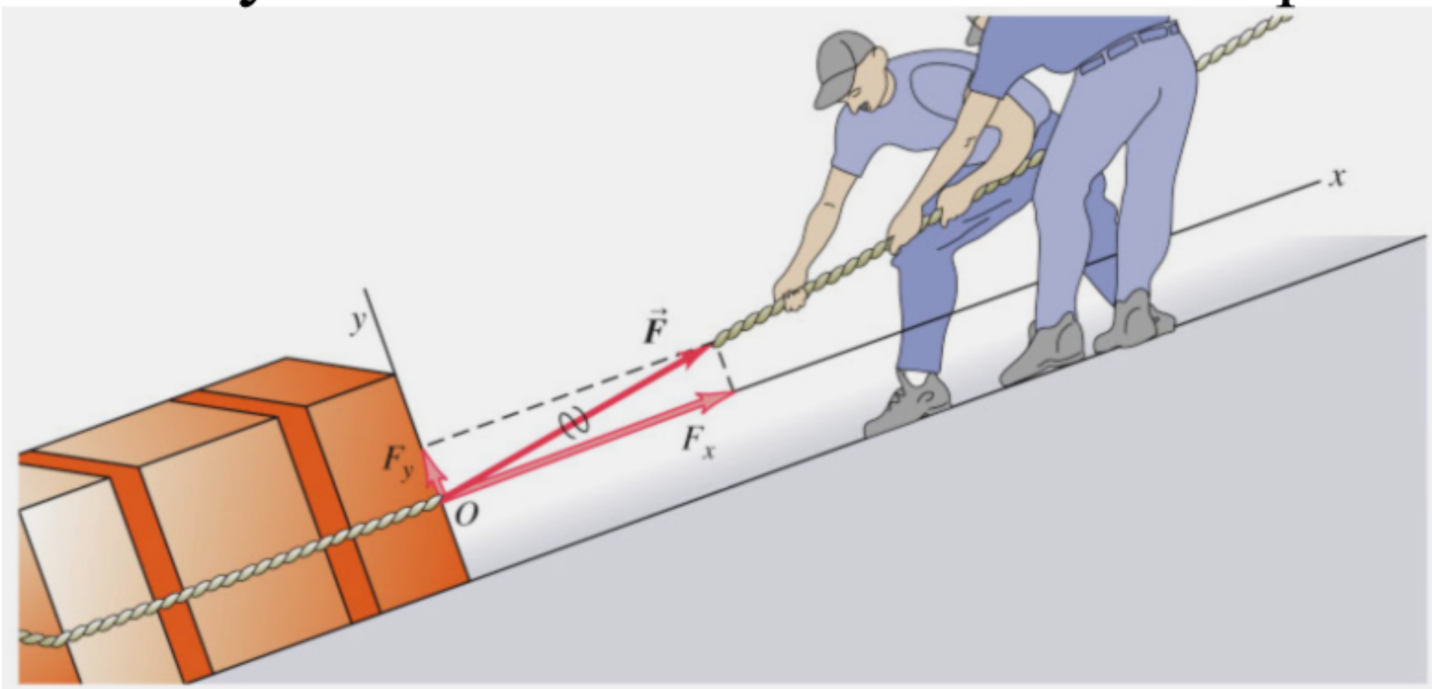
$$\vec{F}_1 = (-5\text{N})\hat{i} \;\; ; \;\; \vec{F}_2 = (5\text{N})\hat{k} \; ; \; \vec{F}_3 = (-5\text{N})\hat{i} + (5\text{N})\hat{k}$$

$$\vec{F}_{net} = \vec{F} + \vec{F}_2 + \vec{F}_3 = 0!$$

Components Of A Force Vector

Superposition of Force principle $\Rightarrow$

Any arbitrary force $\vec{F}$ can be broken into components



$$\vec{F} = \vec{F}_x + \vec{F}_y$$

Pop Quiz:

- All bodies, regardless of mass, accelerate downwards at 9.8 m/s2 at the surface of the earth. This means:
- (A) Gravity's force is independent of mass
- (B) is proportional to mass
- (C) goes as the square of the mass

Weight on Earth & Moon

Standard KG weighs
9.80N on earth

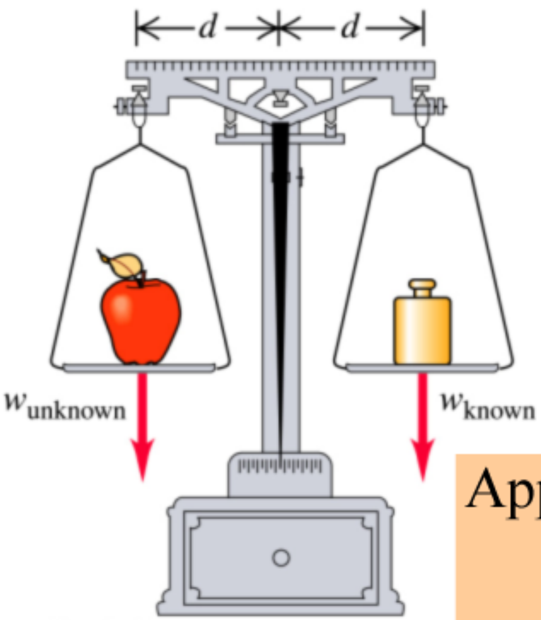
Want to lose
weight?
Go to the moon!

Standard KG weighs
1.6N on moon

Measuring Mass & Weight

On earth, an easy way to measure mass is by measuring its weight and comparing with the standard kg

$$m_1 a_1 = m_2 a_2$$



In Zero gravity you weigh nothing so to measure mass, use 2nd law

Apply a force & measure object's acceleration

$$m = |\vec{F}| / |\vec{a}|$$

A Consequence of The 2nd Law

Large **F** implies large **a** and vice-verca

Human body not capable of handling large F

To stop an accelerating body safely, need to decelerate slowly and over a "long" period of time.

This concept crucial for airbag design, fighter pilot ejector seats, bungee cords & even saving Lois Lane !

Ejection Seat In Fighter Planes

rocket to slowly accelerate pilot vertical to flight direction

used to be explosives, high acc. over short time=BAD