Physics 4A
Lecture 7: Feb. 5, 2015

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UCSD Physics

Momentum

Definition of Force:

$$\sum \vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt}$$

Define new observable : Momentum $\boxed{\vec{p} = m\vec{v}}$

Second Law Reloaded : $\sum \vec{F} = \frac{d\vec{p}}{dt}$

Rapid change in momentum requires LARGE force

but a more gradual change in p requires less force

Center Of Mass Of A System of Particles

"Treat the object as a point particle"

Center of Mass (cm) of a system of particles is the point that moves as though (1) all of system's mass was concentrated there (2) all external forces were applied there

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + ..}{m_1 + m_2 + m_3 + ..} = \frac{\sum_i m_i\vec{r}_i}{\sum_i m_i}$$

r_{cm}= Mass weighted average position of particles

Motion Of Center of Mass of a System

$$\vec{v}_{\rm cm} = \frac{m_1\vec{v}_1 + m_1\vec{v}_1 + m_1\vec{v}_1 ..}{m_1 + m_2 + m_3 + ..} = \frac{\sum_i m_i\vec{v}_i}{M} \quad \Rightarrow \quad \boxed{M\vec{v}_{\rm cm} = \sum_i m_i\vec{v}_i = \vec{P}}$$

Total momentum = total mass x velocity of *center of mass*

$\Rightarrow$ When no net force acts on a system of particles, ***velocity of center of mass*** remains unchanged

Wrench under F $\cong$ 0, spins on a horizontal surface

Center of mass (white dot) moves with $\vec{\mathrm{v}}_{\rm cm} = const$

External Forces & Center of Mass Motion

Define $\vec{a}_{cm} = \frac{d\vec{v}_{cm}}{dt}$

sum of forces on m_2

$$M\vec{a}_{cm} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + ..$$

But $m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + \ldots = \sum \vec{F} = \sum \vec{F}_{ext} + \sum \vec{F}_{\text{int}}$

$$M\vec{a}_{cm} = \sum \vec{F}_{ext} + \sum \vec{F}_{\text{int}} = \sum \vec{F}_{ext} + 0 \Leftarrow \text{3rd law}$$

When system of particles acted upon by external forces, center of mass moves as though all mass was at that point and it were acted upon by a net force = $\sum F_{ext}$

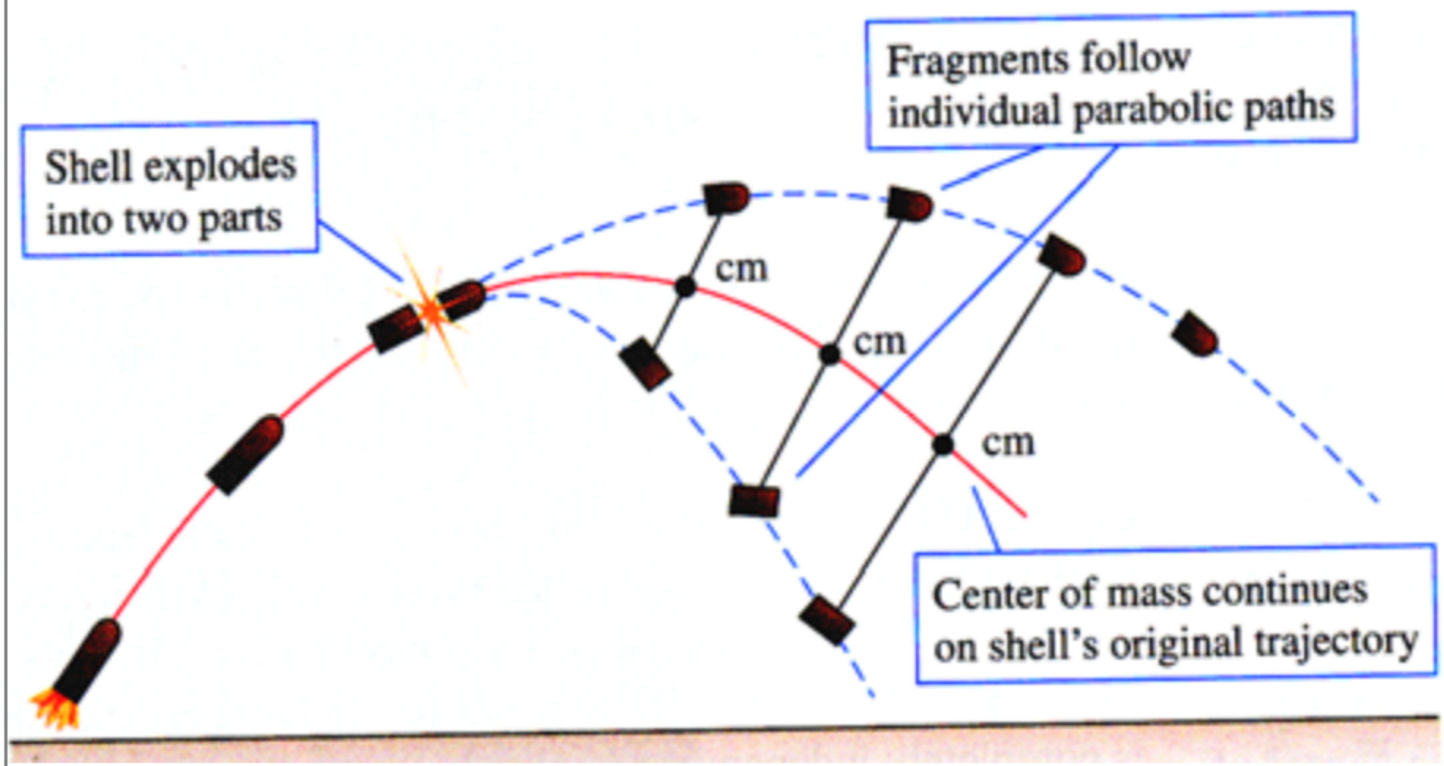
$$M\vec{a}_{cm} = M\frac{d\vec{v}_{cm}}{dt} = \frac{d(M\vec{v}_{cm})}{dt} = \boxed{\frac{d\vec{P}}{dt} = \sum \vec{F}_{ext}}$$

$$\boxed{\text{if } \sum \vec{F}_{ext} = 0 \Rightarrow \vec{P} = \text{constant}}$$

Motion of Center Of Mass Under A Force


Center of mass of diver and baseball follow parabolic path
although all other points follow more complicated path

External Force & CM Motion



Firework !

Impulse Of A Force

If a **constant** net force acts over a time interval Δt

Impulse $\vec{J} = \sum \vec{F} \Delta t$

$$\sum \vec{F} = \frac{\vec{p}_2 - \vec{p}_1}{t_2 - t_1} \Rightarrow$$

$$\sum \vec{F} \Delta t = \vec{p}_2 - \vec{p}_1$$

$$\Rightarrow \text{Impulse } \vec{J} = \vec{p}_2 - \vec{p}_1$$

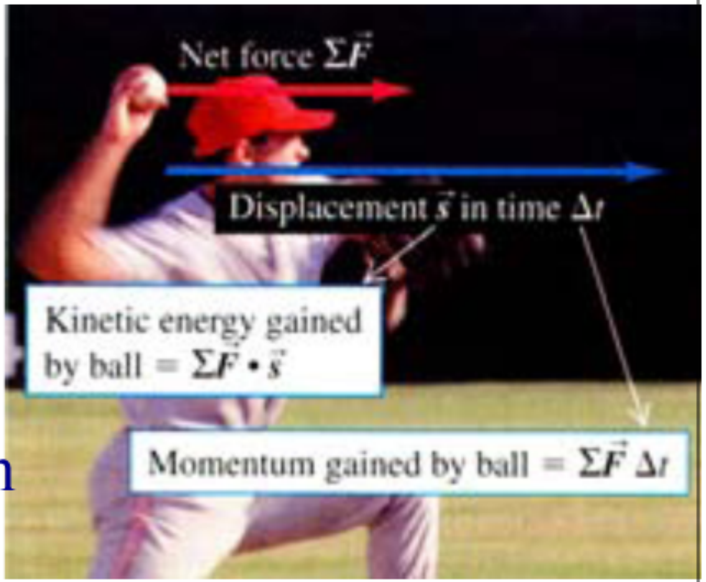
If a variable force acts over time interval Δt

Impulse $\boxed{\vec{J} = \int_{t_1}^{t_2} \sum \vec{F} dt}$

$$\text{Impulse } \vec{J} = \vec{p}_2 - \vec{p}_1 = F_{ave} \Delta t$$

Change in momentum of body during a time Δt equals the impulse of force that acts on body in that time interval

Kinetic Energy, Momentum & Impulse



A body of mass m, initially at rest, is acted upon by a force $\vec{F}$ over a distance $\vec{d}$ such that its final speed is v.

Impulse $\vec{J} =$ gain in momentum

$$= \vec{p}_2 - \vec{p}_1 = \sum \vec{F}\Delta t$$

Kinetic Energy gained

$$E = \sum \vec{F}.\vec{s} = \frac{mv^2}{2} = \frac{p^2}{2m}$$

Quarterback Sack: You Pick !

Mike (m=50kg,v=8m/s) &

Bubba (m=200kg,v=2m/s)

coming to get you!

By whom would, *you*, the quarterback, prefer to be sacked by?

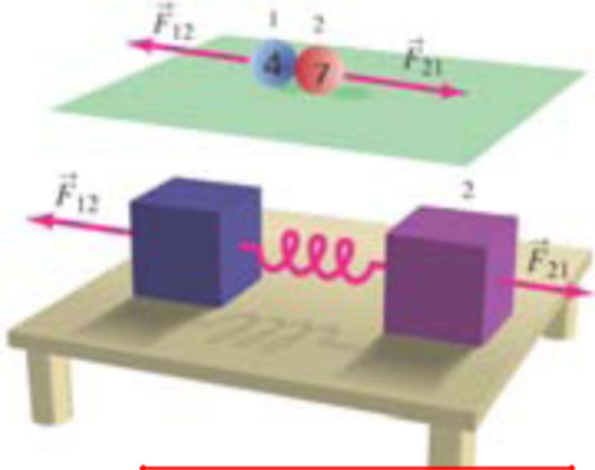
Both have same momentum (p=400kgm/s), will require same impulse Δp to be brought to rest (by you!). But Mike has 4 times the kinetic energy of Bubba ! ($K = mv^2/2$)

For a given force that you exert with your body, it takes same amount of time to stop either guy but your body is pushed back $\times 4$ more by faster guy ! (work done)

To avoid the instantaneous strain on body, Let Bubba get you !

Principle of Conservation Of Momentum

Consider two *isolated* objects interacting with each other



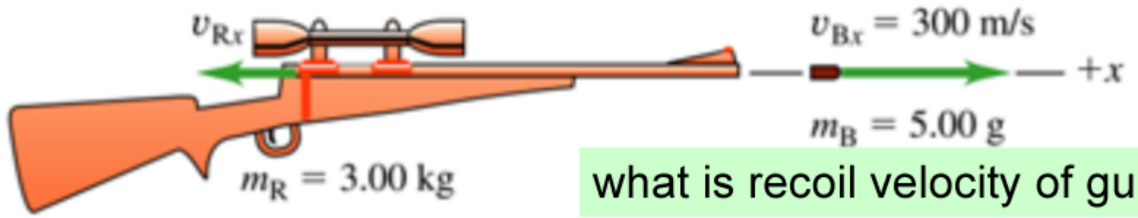
$$\boxed{\vec{F}_{1on2} = -\vec{F}_{2on1}} \Leftrightarrow \boxed{\frac{d\vec{p}_2}{dt} = -\frac{d\vec{p}_1}{dt}}$$

$$\Rightarrow \frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} = 0 \Rightarrow \frac{d(\vec{p}_1 + \vec{p}_2)}{dt} = 0 \Rightarrow \boxed{\vec{p}_1 + \vec{p}_2 = \text{const}}$$

Sum of momenta of an isolated system of objects is constant, no matter what forces act between the objects making up the system

If the vector sum of all *external forces* on a system is zero, then the total momentum of system is constant

Recoil Of A Rifle



what is recoil velocity of gun v_R ?

Assume Hunter+Rifle is an isolated system : $\sum \vec{F} = 0$

$\Rightarrow$ momentum is conserved before & after the shot

Conservation of P before & after $\Rightarrow 0 = m_B v_B + m_R v_R$

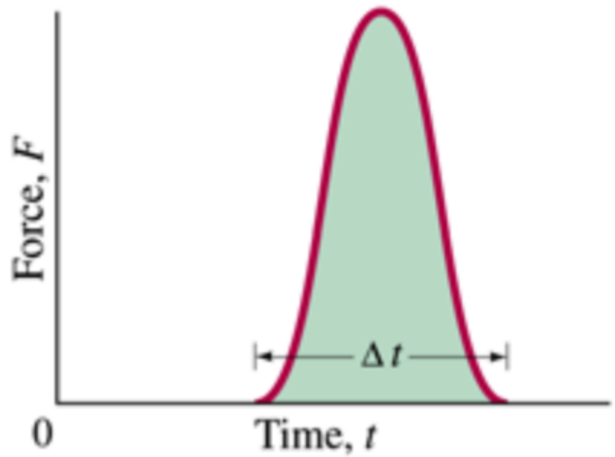
$$\Rightarrow \text{Recoil vel. } v_R = -\frac{m_B}{m_R} v_B = -0.5 m/s, \; p_R = m_R v_R = -1.5 \; kg.m/s$$

$$K_R = \frac{1}{2} m_R v_R^2 = 0.375 \; J; \quad K_B = \frac{1}{2} m_B v_B^2 = 225 J$$

$\Rightarrow$ work done by target to stop the bullet is 600 times larger than that by the hunter to absorb the kinetic energy in Rifle recoil

Physics Of Collision

In collision of two objects, both objects are momentarily or permanently deformed due to large force acting on them. At collision time, the force jumps from zero at the moment of contact to a very large value within a short time, then returns to zero again



Two Types of Collision

Elastic Collision: If forces between interacting bodies are *conservative* then mechanical energy is conserved.
Example: pool balls colliding

Inelastic Collision: Involves non-conservative forces, net mechanical energy after collision is less that before.
Example: Bullet embedding into a block of wood

Completely Inelastic Collision: when colliding bodies stick togather and move as one body after collision

In collision of isolated bodies (net external force =0) momentum is conserved *before* & *after collision*

But only in ELASTIC collision is the ***mechanical energy*** of the system conserved (K1+U1=K2+U2)

Inelastic Collision

In inelastic collision, kinetic energy is not conserved, its transformed into other types of energy such as Thermal or Potential energy $\Rightarrow K_{after} < K_{initial}$

Inverse of an inelastic collision is an **Explosion** where potential energy (chemical or nuclear) is released leading to increase in Kinetic energy of the fragments

Typical macroscopic collisions (like cars crashing) are inelastic. Cars are designed such that kinetic energy before crash is absorbed in the structure of the cars

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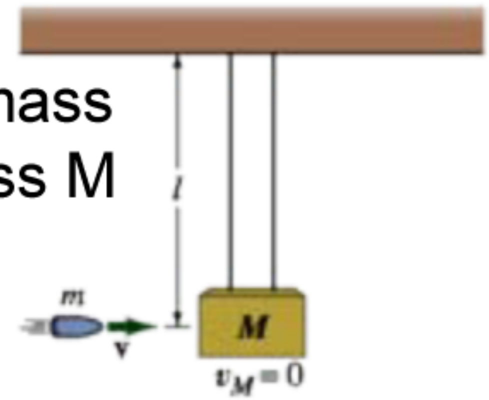
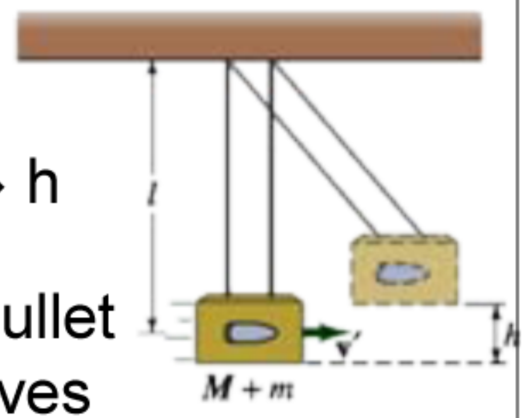
Completely Inelastic Collision: Ballistic Pendulum



Ballistic pendulum is device used to measure speed of bullets. Bullet of mass m fired into pendulum with block of mass M

After collision, block-bullet system swings up to a max. height h.
What is relation between bullet v & h?

Analysis in 2 parts: (a) collision itself
(b) subsequent motion of pendulum → h



(a)Collision happens over short time, bullet comes to rest in block before block moves “too much” from its position at impact.
No net external force ⇒momentum is conserved

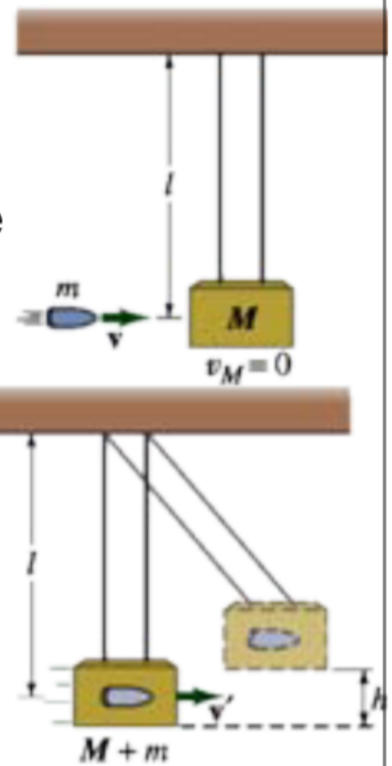
Ballistic Pendulum

(b) after impact, bullet+block moves up

Net external force $\Rightarrow$ gravity pulling system down

Cannot use momentum conservation, but can use

mechanical energy conservation: $K_1+U_1=K_2+U_2$



(Situation a) $\Rightarrow$ mv =(m+M)v'

(Situation b) $\Rightarrow$ Mech. Energy Conservation

$$\frac{1}{2}(m+M)v'^2 + 0 = 0 + (m+M)gh \Rightarrow v' = \sqrt{2gh}$$

$$\text{substituting in (a)} \Rightarrow v = \frac{m+M}{m}v' \,,$$

$$\Rightarrow \mathrm{v} = \frac{m+M}{m}\sqrt{2gh}$$

Can measure bullet speed by watching how much block rises after impact !

Elastic Collision: K & p Are Conserved

E conservation

$$\boxed{\frac{1}{2}m_A v_{A1}^2 + \frac{1}{2}m_B v_{B1}^2 = \frac{1}{2}m_A v_{A2}^2 + \frac{1}{2}m_B v_{B2}^2}$$

p conservation

$$\boxed{m_A v_{A1} + m_B v_{B1} = m_A v_{A2} + m_B v_{B2}}$$

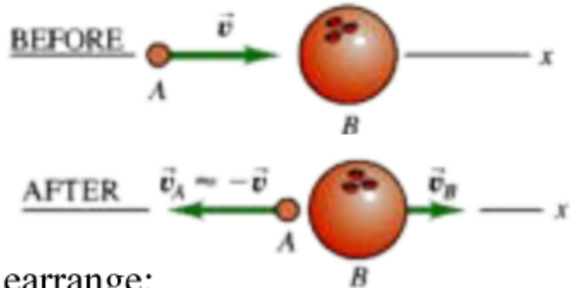
Usually masses and initial velocities are known.
Final velocities obtained from 2 eq. and 2 unknowns

Next, consider some interesting situations

Quick Question

- A pool ball rolling in a straight line with velocity v collides with another stationary ball of the same mass. Neglecting friction and air resistance and assuming an elastic collision which of the following will happen:
- (A) Both balls will move forward together with velocity v/2
- (B) The first ball will bounce back with velocity -v/2 and the second will move forward with velocity v/2
- (C) The first ball will stop and the second ball will move forward with velocity v
- (D) The first ball will bounce back with velocity -v/2 and the second will move forward with velocity 3v/2

Bambi Meets Godzilla & Vice Verca!



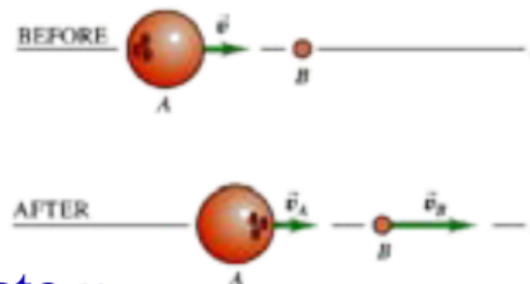
$$\boxed{\frac{1}{2}m_A v^2}_{before} = \boxed{\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2}_{after}$$

$$\boxed{m_A v}_{before} = \boxed{m_A v_A + m_B v_B}_{after}$$

Rearrange:

$$\Rightarrow m_B v_B^2 = m_A(v^2 - v_A^2) = m_A(v - v_A)(v + v_A)$$

$$\& \; m_B v_B \qquad = m_A(v - v_A)$$



dividing 2 eqn $\Rightarrow \boxed{v_B = v + v_A}$

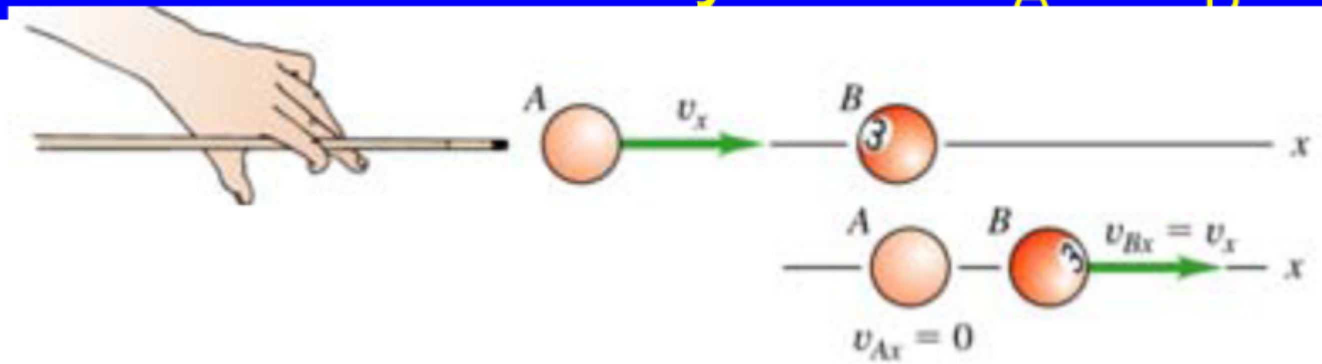
substitute in mom-conserv. eq. to eliminate v_B

$$\Rightarrow m_B(v + v_A) = m_A(v - v_A) \Rightarrow$$

Explain when $m_A \ll m_B$ & $m_A \gg m_B$

$$\boxed{v_A = \frac{m_A - m_B}{m_A + m_B} v} \text{ and } \boxed{v_B = \frac{2m_A}{m_A + m_B} v}$$

Pool Table Physics: $m_A = m_B$

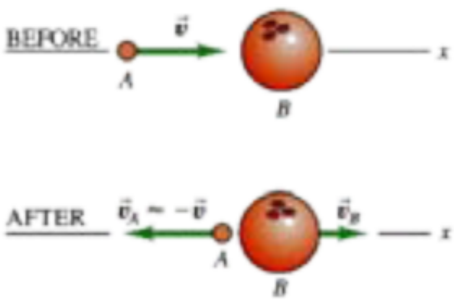


$$\boxed{v_A = \frac{m_A - m_B}{m_A + m_B} v}$$ and $$\boxed{v_B = \frac{2m_A}{m_A + m_B} v}$$

$\Rightarrow$ Final Velocity $v_A = 0$ & $v_B = v$; Striking ball stops & gives up all its momentum & Kinetic energy $\rightarrow$ now at rest

In an elastic collision, relative velocity of the two bodies has the *same* magnitude before & after impact.

Elastic Collision & Relative Velocity



From Energy & momentum conservation we obtained $v_B = v + v_A$

Vel. of B relative to A after collision

= negative of vel. of B relative to A before collision

$$v = v_B - v_A$$

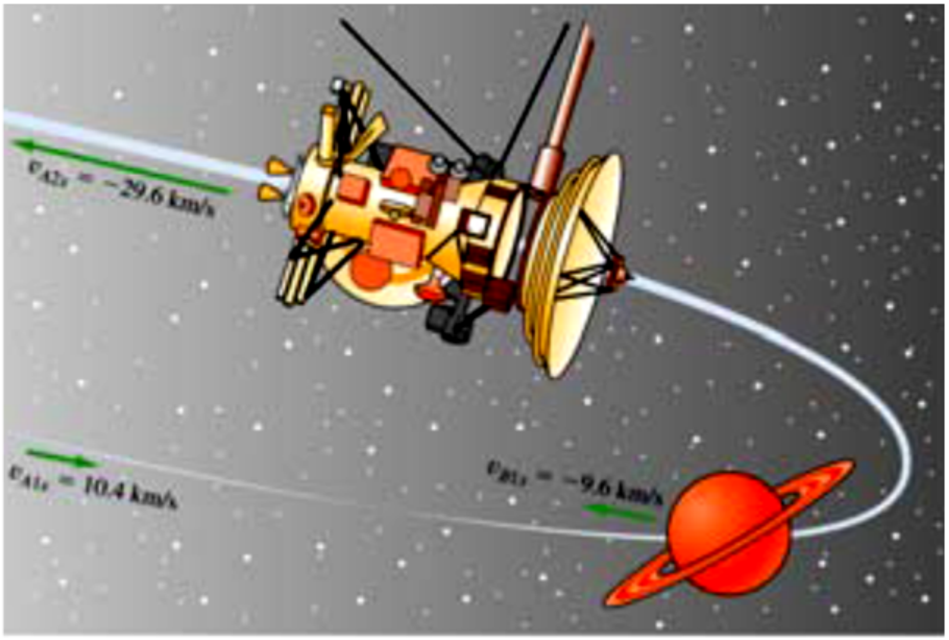
In another Inertial Frame of Reference : A & B have different velocities but their relative velocity is the same

$\Rightarrow$ Relative velocity has same magnitude but opposite direction before and after collision

$$\boxed{v_{B2} - v_{A2} = -(v_{B1} - v_{A1})}$$

Gravitational Slingshot Effect !

Spacecraft with $M_A = 2150kg$ moving with v_{A1}=+10.4km/s (w.r.t sun) approaches Saturn (M_B=5.69×10^{26}kg, moving with v=-9.6m/s). Gravitational attraction of Saturn causes spacecraft to swing around it and head off in oppo direction. Find final speed of spacecraft after it is out of the range of Saturn's gravity.



Collision ? Where?

Grav. Interaction is the same thing

Elastic collision since the interaction force (gravity) is conservative

Craft = A

Saturn = B

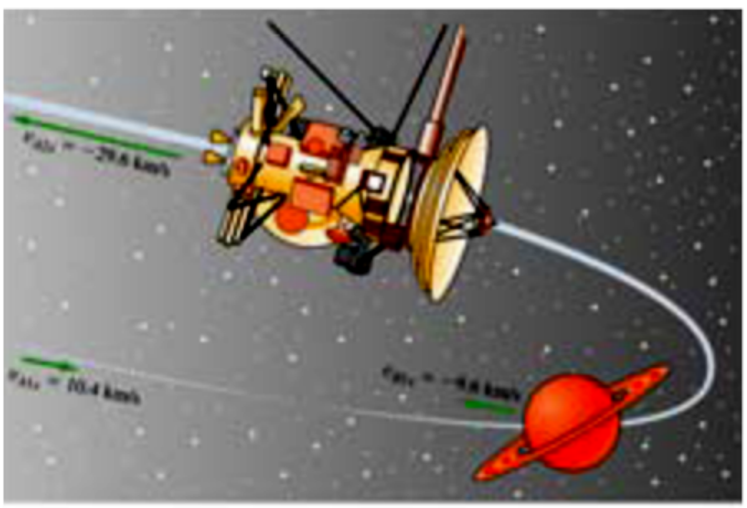
$$\text{Since } \mathrm{M_B} >> \mathrm{M_A} \Rightarrow V_{B2} = V_{B1} = -9.6 km/s$$

Take Craft's original direction as along +x.

Rel. Vel: *Use* $\boxed{v_{B2} - v_{A2} = -(v_{B1} - v_{A1})}$:Elastic collision

$$\Rightarrow V_{A2} = V_{B2} + V_{B1} - V_{A1}$$

$$=[(-9.6)+(-9.6)-10.4]\text{km/s}=-29.6\text{km/s}$$



Craft's speed is x3 larger after "collision"

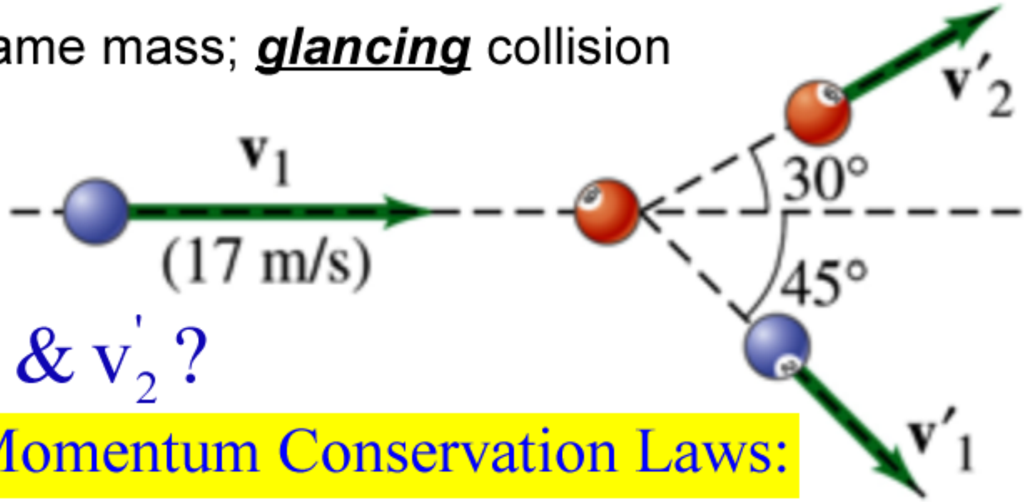
& Craft's Kinetic energy is x 8.1 larger after interaction !

Grav. slingshot effect !

like ball being hit by a swinging baseball→homerun

Elastic Collision of Pool Balls in 2D

Both balls have same mass; ***glancing*** collision



What are $v_1^{'}$ & $v_2^{'}$?

Use Energy & Momentum Conservation Laws:

$$\text{K Energy: } \frac{1}{2}mv_1^2 + 0 = \frac{1}{2}mv'^2_1 + \frac{1}{2}mv'^2_2$$

$$\sum P_x \Rightarrow: \mathrm{mv}_1 + 0 = \mathrm{mv}_{1x}^{'} + \mathrm{mv}_{2x}^{'} = \mathrm{mv}_1^{'} \cos 45 + \mathrm{mv}_2^{'} \cos 30$$

$$\sum P_y \Rightarrow \; 0 + 0 = \mathrm{mv}_{1y}^{'} + \mathrm{mv}_{2y}^{'} = m\mathrm{v}_1^{'} \sin 45 + \mathrm{mv}_2^{'} \sin 30$$

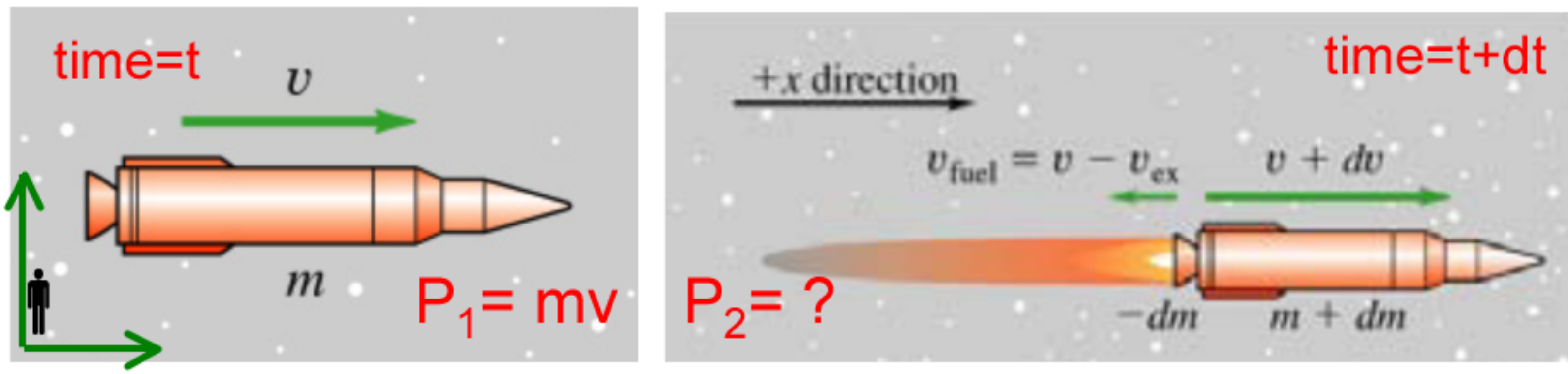
Solve for $v_1^{'}$ & $v_2^{'}$ using last 2 eqns.

Rocket & Jet Propulsion

Rocket or Jet Plane: System's mass changes with time as burnt fuel mass expelled

Rocket Propulsion In Absence Of Gravity

Consider Rocket as isolated system in space, no gravity



In time dt, rocket ejects burnt mass -dm with rel. velocity v_{ex}

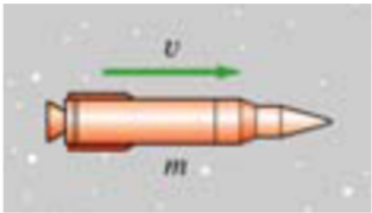
According to observer, $v_{fuel} = v - v_{ex}$ & $P_{fuel} = (-dm)(v - v_{ex})$

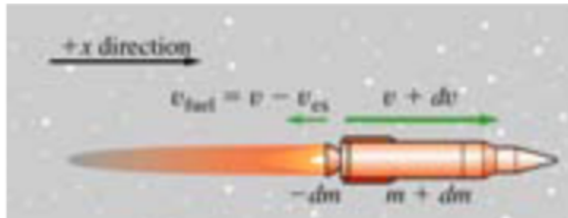
after time dt, rocket's mass $\boxed{m \rightarrow m+dm}$; $\boxed{v \rightarrow v+dv}$

rocket's momentum now is $P_r = (m+dm)(v+dv)$

Total mom. $P_2 = P_r + P_{fuel} = (m+dm)(v+dv) + (-dm)(v - v_{ex})$

Rocket Propulsion In Absence Of Gravity





Rocket = isolated
No net force on it
Momentum conserved
$\Rightarrow$ **$P_1 = P_2$**

$$mv = (m+dm)(v+dv) + (-dm)(v - v_{ex}) \Rightarrow m dv = -dm\ v - \cancel{dm\,dv}$$

$$\Rightarrow m\frac{dv}{dt} = -v_{ex}\frac{dm}{dt} \Rightarrow ma = \boxed{F_{thrust} = -v_{ex}\frac{dm}{dt}}$$

$$\boxed{a_{rocket} = -\frac{v_{ex}}{m}\frac{dm}{dt} > 0 \text{ since } dm < 0}$$

If v_{ex} & $\frac{dm}{dt}$ are held constant then rocket acceleration increases till all fuel is gone !

Rocket Launched Under Gravity: $F_{ext} = mg$

$$a_{rocket} = \frac{dv}{dt} = \frac{F_{ext}}{m} - \frac{v_{ex}}{m}\frac{dm}{dt}$$

$$\Rightarrow \int_{v_0}^{v} dv' = \int_{t=0}^{t} \frac{F_{ext}}{m} dt - v_{ex} \int_{m_0}^{m} \frac{dm'}{m'}$$

$$\Rightarrow \boxed{v - v_0 = -gt + v_{ex} \ln \frac{m_0}{m}}$$

Velocity change of Rocket fired from Earth

A “Rocket like” Problem

What happens to Boat’s motion after throwing package ?