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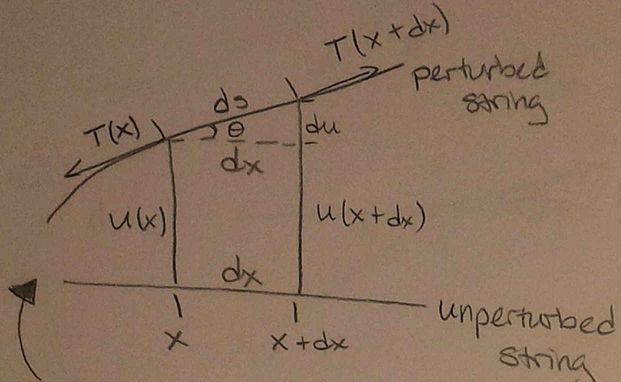
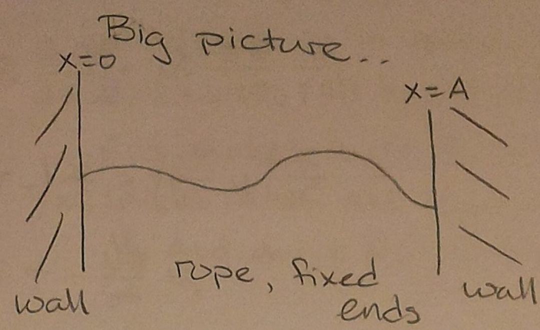
Phys 200A
Pset 2

1.



small section of rope.



$\delta u(x=0) = \delta(x=A) = 0$: fixed ends

a) $ds^2 = du^2 + dx^2$, given T (tension) and mass density μ.

The variable $u(x,t)$ is the vertical displacement of the rope section. x and t are parameters.

For the small section:

$$KE = \frac{1}{2}(\mu\, dx)\left(\frac{\partial u}{\partial t}\right)^2 = \frac{1}{2}\mu\left(\frac{\partial u}{\partial t}\right)^2 dx$$

PE : change of tension from unperturbed to perturbed. $dW = T\,ds - T\,dx$

$$PE = T(ds - dx) = T\left([du^2 + dx^2]^{1/2} - dx\right) = T\left(\left[\left(\frac{du}{dx}\right)^2 + 1\right]^{1/2} dx - dx\right) = T\left(\left[\left(\frac{du}{dx}\right)^2 + 1\right]^{1/2} - 1\right) dx$$

assume small changes in $\frac{du}{dx}$ so we can Taylor expand $\left[1 + \left(\frac{du}{dx}\right)^2\right]^{1/2} \approx 1 + \frac{1}{2}\left[\frac{du}{dx}\right]^2 - \dots$

$PE = T\left(1 + \frac{1}{2}\left(\frac{du}{dx}\right)^2 - 1\right)dx = T\frac{1}{2}\left(\frac{du}{dx}\right)^2 dx$ and since x and t are independent

$\frac{du}{dx} = \frac{\partial u}{\partial x}$ so $PE = \frac{T}{2}\left(\frac{\partial u}{\partial x}\right)^2 dx$

$$\mathcal{L} = KE - PE = \frac{\mu}{2}\left(\frac{\partial u}{\partial t}\right)^2 dx - \frac{T}{2}\left(\frac{\partial u}{\partial x}\right)^2 dx$$: Lagrangian density

so $L = \int_0^A \mathcal{L}\, dx = \underline{\int_0^A \left[\frac{\mu}{2}\left(\frac{\partial u}{\partial t}\right)^2 - \frac{T}{2}\left(\frac{\partial u}{\partial x}\right)^2\right] dx}$ ⟵ Lagrangian

$$S = \int_{t_1}^{t_2} L\, dt = \int_{t_1}^{t_2} dt \int_0^A \mathcal{L}\, dx$$

$\delta S = \int_{t_1}^{t_2} dt \int_0^A \left(\frac{\partial \mathcal{L}}{\partial u_t}\delta u_t + \frac{\partial \mathcal{L}}{\partial u_x}\delta u_x\right) dx$ and since the ends are fixed, $\delta S = 0$

and for $\delta S = 0$, the inside of the integral is zero

meaning: $\frac{\partial \mathcal{L}}{\partial u_t} \delta u_t + \frac{\partial \mathcal{L}}{\partial u_x} \delta u_x = 0$

$\delta u_t = \frac{\partial}{\partial t} \delta u$ and $\delta u_x = \frac{\partial}{\partial x} \delta u$ so then

$$\frac{\partial \mathcal{L}}{\partial u_t} \frac{\partial}{\partial t} \delta u + \frac{\partial \mathcal{L}}{\partial u_x} \frac{\partial}{\partial x} \delta u = 0$$

$$\frac{\partial}{\partial t} \frac{\partial \mathcal{L}}{\partial u_t} \delta u + \frac{\partial}{\partial x} \frac{\partial \mathcal{L}}{\partial u_x} \delta u = 0$$

$$\frac{\partial}{\partial t}\left(\mu \frac{\partial u}{\partial t}\right) + \frac{\partial}{\partial x}\left(-T \frac{\partial u}{\partial x}\right) = 0$$

$$\mu \frac{\partial^2 u}{\partial t^2} = T \frac{\partial^2 u}{\partial x^2}$$

$$\frac{\mu}{T} \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} \quad \text{— LEOM}$$

let $\frac{\mu}{T} = \frac{1}{c^2}$ so then

$$\frac{1}{c^2}\left(\frac{\partial^2 u}{\partial t^2}\right) = \frac{\partial^2 u}{\partial x^2}$$ = wave equation with assumption of $\frac{\partial u}{\partial x}$ is small.

b) $\mathcal{H} = p_i \dot{q}_i - \mathcal{L}$ where p_i is a conjugate momentum $p_i = \frac{\partial \mathcal{L}}{\partial \dot{q}_i}$ for each generalized velocity. In this problem we only have one generalized velocity. $\frac{\partial u}{\partial t}$

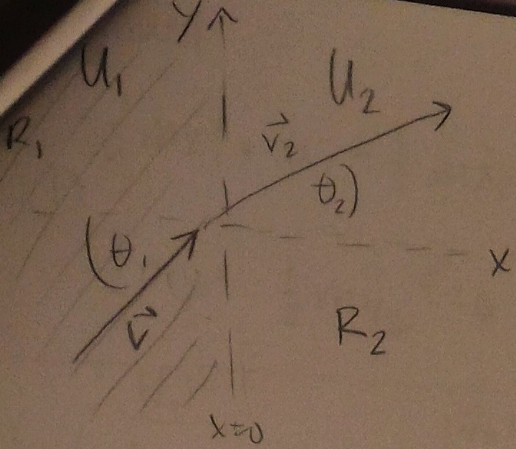
so $P_t = \frac{\partial \mathcal{L}}{\partial u_t} = \mu u_t \longrightarrow \mathcal{H} = \mu u_t^2 - \mathcal{L} = \mu u_t^2 - \frac{\mu}{2} u_t^2 + \frac{T}{2} u_x^2$

$$= \frac{\mu}{2} u_t^2 + \frac{T}{2} u_x^2$$

$$\underline{H = \int_0^A \left[\frac{\mu}{2}\left(\frac{\partial u}{\partial t}\right)^2 + \frac{T}{2}\left(\frac{\partial u}{\partial x}\right)^2 \right] dx} = \int_0^A \left[\frac{P_t^2}{2\mu} + \frac{T}{2}\left(\frac{\partial u}{\partial x}\right)^2 \right] dx$$

$$\frac{\partial \mathcal{H}}{\partial P_t} = \frac{\partial u}{\partial t} = \frac{P_t}{\mu} \qquad \dot{P}_t = -\frac{\partial H}{\partial u}$$

$$\underset{\substack{\uparrow \\ \text{What we found above}}}{\mu u_t = P_t}$$



a. $L = T - U \qquad T_1 = \frac{1}{2}m|\vec{v}_1|^2, \; T_2 = \frac{1}{2}m|\vec{v}_2|^2$

$$U_1 = U_1, \; U_2 = U_2$$

$$\therefore L = \frac{1}{2}m[\dot{x}^2 + \dot{y}^2] - U(x)$$

where $U(x) = \begin{cases} U_1 & x<0 \\ U_2 & x>0 \end{cases}$

$$\begin{cases} x: \dfrac{\partial \mathcal{L}}{\partial x} = \dfrac{d}{dt}\dfrac{\partial \mathcal{L}}{\partial \dot{x}} \\ -\dfrac{\partial U}{\partial x} = \dfrac{d}{dt}(m\dot{x}) = m\ddot{x} \end{cases} \rightarrow m\ddot{x} = -\frac{\partial U}{\partial x} \rightarrow \boxed{m\ddot{x} + \frac{\partial U}{\partial x} = 0} \quad \begin{cases} p = m\dot{x} \\ \dot{x} = \dfrac{p}{m} = \dfrac{dx}{dt} \end{cases}$$

* need to get into terms of ps & qs *

$$m\ddot{x} = \frac{d}{dt}(\underbrace{m\dot{x}}_{p}) = \frac{dP_x}{dt} = \frac{dP_x}{dx}\cdot\frac{dx}{dt} = \frac{P_x}{m}\frac{dP_x}{dx}$$

$$\begin{cases} y: \dfrac{\partial \mathcal{L}}{\partial y} = \dfrac{d}{dt}\dfrac{\partial \mathcal{L}}{\partial \dot{y}} \\ 0 = \dfrac{d}{dt}(m\dot{y}) = \boxed{m\ddot{y} = 0} \end{cases}$$

NOW: $\left(\frac{P_x}{m}\cdot\frac{dP_x}{dx} + \frac{dU}{dx}\right)dx = 0\,dx$

$$\int_{R_1}^{R_2}\frac{P_x}{m}\frac{dP_x}{dx}dx + \int_{R_1}^{R_2}\frac{dU}{dx}dx = \int_{P_1}^{P_2}\frac{P_x}{m}dP_x + \int_{U_1}^{U_2}U(x) = \left[\frac{P_{x_2}^2}{2m} - \frac{P_{x_1}^2}{2m}\right] + U_2 - U_1 = 0$$

$$\frac{(m\dot{x})^2}{2m} = \frac{1}{2}m\dot{x}^2$$

So we get conservation of Energy!

$$\frac{1}{2}m\dot{x}_1^2 + U_1 = \frac{1}{2}m\dot{x}_2^2 + U_2$$

time for y: $m\ddot{y} = 0 \;\therefore\; m\dot{y} = \text{constant}!$ Momentum in y direction = constant

$m\dot{y}_1 = m\dot{y}_2$ ✓ in terms of ps & ys

$$\int_{R_1}^{R_2}\frac{P_y}{m}\frac{dP_y}{dy}dy = 0 \Rightarrow \int_{P_{y_1}}^{P_{y_2}}\frac{P_y}{m}dP_y \Rightarrow \frac{P_{y_2}^2}{2m} - \frac{P_{y_1}^2}{2m} = 0 ✓$$

Energy is the same as well!

$$\left\{\frac{1}{2}m\dot{y}_1^2 = \frac{1}{2}m\dot{y}_2^2\right\}$$

...inued

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$$\frac{1}{2}m\dot{x}_1^2 + U_1 - \frac{1}{2}m\dot{x}_2^2 - U_2 = 0$$

$+$

y: $$\frac{1}{2}m\dot{y}_1^2 - \frac{1}{2}m\dot{y}_2^2 = 0$$

$$\frac{1}{2}m\underbrace{(\dot{x}_1^2 + \dot{y}_1^2)}_{|\vec{v}_1|^2} - \frac{1}{2}m\underbrace{(\dot{x}_2^2 + \dot{y}_2^2)}_{|\vec{v}_2|^2} + U_1 - U_2 = 0$$

Region 1 $\longrightarrow$ Region 2

$$\frac{1}{2}m|\vec{v}_1|^2 + U_1 = \frac{1}{2}m|\vec{v}_2|^2 + U_2$$

note

$$\boxed{\begin{aligned}\dot{x} &= v\cos\theta \\ \dot{y} &= v\sin\theta\end{aligned}}$$

Recall from $\dot{y}$ $\quad m\dot{y}_1 = m\dot{y}_2 \Rightarrow \cancel{m}v_1\sin\theta_1 = \cancel{m}v_2\sin\theta_2$

$$\therefore \frac{v_1}{v_2} = \frac{\sin\theta_2}{\sin\theta_1} \longrightarrow v_2 = \frac{\sin\theta_1}{\sin\theta_2}v_1$$

we know that v_2 also equals:

$$v_2 = \sqrt{\frac{\frac{1}{2}mv_1^2 + (U_1 - U_2)}{\frac{1}{2}m}}\left(\frac{v_1}{v_1}\right) = \sqrt{\frac{\frac{1}{2}mv_1^2 + (U_1 - U_2)}{\underbrace{\frac{1}{2}mv_1^2}_{T_1}}}\, v_1$$

$$v_2 = \sqrt{1 + \frac{(U_1 - U_2)}{T_1}}\, v_1$$

solving for θ_2:

$$\sin\theta_2 = \sin\theta_1 \frac{v_1}{v_2} \Rightarrow \theta_2 = \sin^{-1}\left[\left(\sqrt{1 + \frac{(U_1 - U_2)}{T_1}}\right)^{-1}\sin\theta_1\right]$$

ratio of times; diff mass same U

masses are on the same path → conservation of energy

- Potential energy is the same → kinetic energy must be constant

let $m \to bm$ & $t \to ct$, where c & b are constants

$$\frac{1}{2}m_1v_1^2 = \frac{1}{2}m_1\frac{x^2}{t_1^2} = \underbrace{\frac{1}{2}bm_1\frac{x^2}{c^2t_1^2} = \frac{1}{2}bm_2\frac{x^2}{c^2t_2^2}}$$

★ note

$$\boxed{\frac{1}{2}m_1v_1^2 + \cancel{U} = \frac{1}{2}m_2v_2^2 + \cancel{U}}$$

equal & constant

$$\frac{m_1}{\cancel{c^2}t_1^2} = \frac{m_2}{\cancel{c^2}t_2^2}$$

$$\frac{t_1^2}{t_2^2} = \frac{m_1}{m_2} \Rightarrow \frac{t_1}{t_2} = \sqrt{\frac{m_1}{m_2}}$$

OR

$$\frac{t_2}{t_1} = \sqrt{\frac{m_2}{m_1}}$$

(c) same mass diff U_1 & U_2 ratio of times

given: $\frac{U_2}{U_1} = K$ $\quad U_2 \to KU_1$, $\quad t \to ct$

Kinetic energy = constant b/c total energy is still constant

$$\frac{1}{2}mv^2 = \frac{1}{2}m\frac{x^2}{t^2} = \frac{1}{2}m\frac{x^2}{c^2t^2} \to \frac{1}{2}m\frac{x^2}{c^2t_1^2} + U_1 = \frac{1}{2}m\frac{x^2}{c^2t_2^2} + U_1K$$

$$\underbrace{\frac{1}{2}m\frac{x^2}{c^2t_1^2}}_{\propto KU} = \frac{1}{2}m\frac{x^2}{c^2t_2^2} + U_1(K-1)$$

$K \propto \frac{1}{c^2}$ $\quad c \propto \frac{1}{\sqrt{K}} \propto \sqrt{\frac{U_1}{U_2}}$

so $$\frac{t_2}{t_1} = \left(\frac{U_2}{U_1}\right)^{-1/2}$$

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$$L = -mc^2(1 - v^2/c^2)^{1/2} - V(\vec{r})$$
$$\vec{v} = \dot{\vec{r}}$$

) $$\frac{d}{dt}\left(\frac{\partial L}{\partial \vec{v}}\right) = \frac{\partial L}{\partial \vec{r}}$$

$$\frac{d}{dt}\left[\frac{mc^2 \vec{v}}{c^2\sqrt{1-v^2/c^2}}\right] = -\frac{\partial V}{\partial \vec{r}}$$

$$m\underbrace{\frac{d}{dt}\left[\frac{\vec{v}}{\sqrt{1-v^2/c^2}}\right]}_{\text{acceleration}} = -\frac{\partial V}{\partial \vec{r}} \longrightarrow \vec{F}(\vec{r}) = -\frac{\partial V}{\partial \vec{r}}$$

If $v << c$, $\vec{F}(\vec{r}) = m\frac{d}{dt}(\vec{v}) = m\vec{a}$.
For the non-relativistic case, we recover the classical equation.

) $$\vec{p} = \frac{\partial L}{\partial \dot{\vec{r}}} = \frac{\partial L}{\partial \vec{v}} = \frac{m\vec{v}}{\sqrt{1-v^2/c^2}}$$

o find $\vec{v}$ in terms of $\vec{p}$:

$$m\vec{v} = \vec{p}\sqrt{1-v^2/c^2}$$
$$m^2v^2 = p^2(1-v^2/c^2) = p^2 - p^2v^2/c^2$$
$$m^2v^2 + p^2v^2/c^2 = p^2$$
$$v^2(m^2 + p^2/c^2) = p^2$$
$$v^2 = \frac{p^2}{m^2 + p^2/c^2}$$
$$\frac{v^2}{c^2} = \frac{p^2}{m^2c^2 + p^2}$$

plug this in

To find the Hamiltonian:

$$H = \vec{p}\cdot\vec{v} - (-mc^2(1-v^2/c^2)^{1/2} - V(\vec{r}))$$
$$= \frac{mv^2}{(1-v^2/c^2)^{1/2}} + mc^2(1-v^2/c^2)^{1/2} + V(\vec{r})$$
$$= \frac{mv^2}{(1-v^2/c^2)^{1/2}} + \frac{mc^2(1-v^2/c^2)}{(1-v^2/c^2)^{1/2}} + V(\vec{r})$$
$$= \frac{\cancel{mv^2} + mc^2 - \cancel{mv^2c^2/c^2}}{(1-v^2/c^2)^{1/2}} + V(\vec{r})$$
$$= \frac{mc^2}{(1-v^2/c^2)} + V(\vec{r})$$
$$= \frac{mc^2}{(1-p^2/m^2c^2+p^2)^{1/2}} + V(\vec{r})$$
$$= \frac{mc^2(m^2c^2+p^2)^{1/2}}{(m^2c^2+\cancel{p^2}-\cancel{p^2})^{1/2}} + V(\vec{r})$$
$$= \frac{\cancel{m}c\cancel{^2}(m^2c^2+p^2)^{1/2}}{\cancel{m}\cancel{c}} + V(\vec{r})$$
$$= c(m^2c^2+p^2)^{1/2} + V(\vec{r})$$
$$H = (m^2c^4 + p^2c^2)^{1/2} + V(\vec{r}).$$

he Lagrangian is
$$L = -mc^2(1-v^2/c^2)^{1/2} - V(\vec{r})$$

This does not explicitly depend on time, therefore the Hamiltonian is a constant of motion.

ntinued.

$V \longrightarrow V(r)$ for a spherically symmetric potential

$$\dot{\vec{P}} = -\vec{\nabla} V = -\frac{\partial}{\partial r} V(\vec{r}) \quad \text{and} \quad \vec{P} = \gamma m \dot{\vec{r}}$$

To check if $\vec{r} \times \vec{p}$ is a constant of motion, $\frac{\partial}{\partial t}(\vec{r} \times \vec{p}) \overset{!}{=} 0$.

$$\frac{\partial}{\partial t}(\vec{r} \times \vec{P}) = \frac{\partial}{\partial t}\vec{r} \times \vec{P} + \vec{r} \times \frac{\partial}{\partial t}\vec{P}$$

$$= \dot{\vec{r}} \times \vec{p} + \vec{r} \times \dot{\vec{P}}$$

$$= \gamma m \cancel{(\dot{\vec{r}} \times \dot{\vec{r}})}^{\to 0} + \cancel{\vec{r} \times \left(-\frac{\partial}{\partial r} V(\vec{r})\right)}^{\to 0}$$

Cross product of // vectors = 0.

$= \underline{0} \longrightarrow$ This means $\vec{r} \times \vec{p}$ is a constant of motion

For this spherical symmetry, $P_\theta = 0$, so $\theta = \frac{\pi}{2}$.

$$p^2 = P_r^2 + \cancel{\frac{P_\theta^2}{r^2}}^{\to 0} + \frac{P_\phi^2}{r^2 \cancel{\sin^2\theta}^{\to 1}}$$

$$= P_r^2 + \frac{P_\phi^2}{r^2}$$

From part b,

$$H = (m^2c^4 + p^2c^2)^{1/2} + V(r)$$

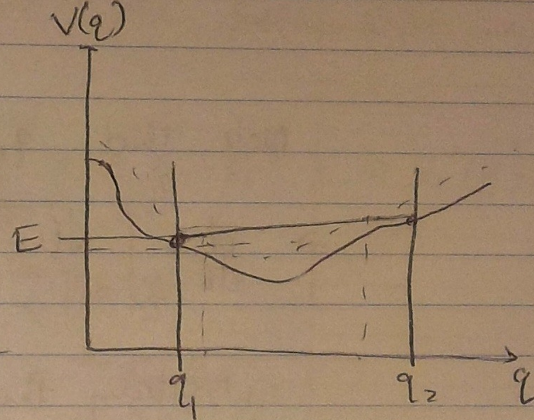
$$= c(m^2c^2 + p^2)^{1/2} + V(r)$$

$$= c\left(m^2c^2 + P_r^2 + \frac{P_\phi^2}{r^2}\right)^{1/2} + V(r)$$

7. $H(q,p) = \frac{p^2}{2m} + V(q)$

$\because \partial_t H = 0$, system is conservative.

let total energy of system be E.



$$\frac{p^2}{2m} + V(q) = E \longrightarrow (1)$$

q_1 & q_2 are the limits of the motion, such that $V(q_1) = V(q_2) = E$.

So, q_1 & q_2 are function of E for given potential function.

The hamiltonian equations of motion are :

$$\dot{p} = -\frac{\partial V}{\partial q}, \quad \dot{q} = \frac{p}{m}$$

From (1), $p = \sqrt{2m(E-V)}$

$$\Rightarrow \frac{dq}{dt} = \sqrt{\frac{2}{m}} \sqrt{(E-V)}$$

$$\Rightarrow \frac{T}{2} = \int_{q_1}^{q_2} \sqrt{\frac{m}{2}} \left(\sqrt{(E-V)}\right)^{-1} dq$$ where T is the time period of oscillation.

$$\Rightarrow T = \sqrt{2m} \int_{q_1}^{q_2} \left(\sqrt{(E-V)}\right)^{-1} dq.$$

Now consider,

$$\frac{d}{dE} \int_{q_1}^{q_2} \left(\sqrt{E-V}\right) dq$$

Using Leibniz integral rule,

$$\frac{d}{dE}\int_{q_1}^{q_2}\left(\sqrt{E-V}\right)dq = \int_{q_1}^{q_2}\frac{dq}{2\sqrt{E-V}} + \sqrt{E-V(q_2)}\,\frac{dq_2}{dE}$$

$$- \sqrt{E-V(q_1)}\,\frac{dq_1}{dE}.$$

Note that q_1 & q_2 are defined such that $V(q_1) = V(q_2) =$

$$\Rightarrow \frac{d}{dE}\int_{q_1}^{q_2}\sqrt{(E-V)}\,dq = \frac{1}{2}\int_{q_1}^{q_2}\frac{dq}{\sqrt{E-V}}.$$

Substituting this into the expression for time period,

$$T = 2\sqrt{2m}\,\frac{d}{dE}\int_{q_1}^{q_2} dq\,\sqrt{E-V}$$

For some potential function $V_0(q)$,

$$T_0(E) = 2\sqrt{2m}\,\frac{d}{dE}\int_{q_1^0}^{q_2^0} dq\,\sqrt{E-V_0}$$

$$V_0(q_1^0) = V_0(q_2^0) = 0.$$

We consider a small perturbation to V_0,

$$V_0 \longrightarrow V_0(q) + \epsilon V_1(q), \quad \epsilon \longrightarrow 0$$

The limits of motion then change to, q_1 & q_2 s.t.

$$E = V_0(q_1) + \epsilon V_1(q_1) = V_0(q_2) + \epsilon V_1(q_2)$$

$q_1 = q_1^0 + \delta q_1$ $\qquad q_2 = q_2^0 + \delta q_2$

Then

$$V_0(q_1^0) = V_0(q_1^0 + \delta q_1) + \epsilon V_1(q_1^0 + \delta q_1)$$

Assuming the potentials are well behaved & the perturbation is not too large,

$$V_0(q_1^0) = V_0(q_1^0) + V_0'(q_1^0)\,\delta q_1 + \epsilon V_1(q_1^0) + \epsilon V_1'(q_1^0)\,\delta q_1$$

Then to a first order approximation

$$\delta q_1 = -\epsilon \frac{V_1(q_1^0)}{V_0'(q_1^0)} = \frac{-\epsilon E}{V_0'(q_1^0)}$$

Similarly $\quad \delta q_2 = -\epsilon \dfrac{V_1(q_2^0)}{V_0'(q_2^0)} = \dfrac{-\epsilon E}{V_0'(q_2^0)}$

Now,

$$T(E) = 2\sqrt{2m}\,\frac{d}{dE}\int_{q_1}^{q_2} dq\,\sqrt{E - V_0 - \epsilon V_1}$$

$$\Rightarrow T(E) = 2\sqrt{2m}\,\frac{d}{dE}\left\{\left[\int_{q_1}^{q_1^0} + \int_{q_1^0}^{q_2^0} + \int_{q_2^0}^{q_2}\right] dq\sqrt{E - V_0 - \epsilon V_1}\right\}$$

$\because$ ϵ is small, the integrals can be reduced to

$$T(E) = 2\sqrt{2m}\,\frac{d}{dE}\left\{\left(\sqrt{E - V_0(q_1^0) - \epsilon V_1(q_1^0)}\right)\frac{\epsilon E}{V_0'(q_1^0)} + \frac{-\epsilon E\sqrt{E - V_0(q_2^0) - \epsilon V_1(q_2^0)}}{V_0'(q_2^0)} + \int_{q_1^0}^{q_2^0} dq\,\sqrt{E - V_0 - \epsilon V_1}\right.$$

$$\Rightarrow T(E) = 2\sqrt{2m}\,\frac{d}{dE}\left[\left(\sqrt{-V_1(q_1^0)\epsilon}\right)\frac{E\epsilon}{V_0'(q_1^0)} - \frac{E\epsilon}{V_0'(q_2^0)}\sqrt{-\epsilon V_1(q_2^0)} + \int_{q_1^0}^{q_2^0} dq\,\sqrt{E - V_0 - \epsilon V_1}\right]$$

Note that the first two terms are of order $\epsilon^{1.5}$. So we neglect those terms to get

$$T = 2\sqrt{2m}\,\frac{d}{dE}\int_{q_1^0}^{q_2^0} dq\,\sqrt{E-V_0(q)}\left[1-\frac{\epsilon V_1(q)}{E-V_0(q)}\right]$$

$\because$ ϵ is small, $\left(1-\frac{\epsilon V_1}{E-V_0}\right)^{1/2} \approx 1-\frac{1}{2}\frac{\epsilon V_1}{E-V_0}$

$$\Rightarrow T = 2\sqrt{2m}\,\frac{d}{dE}\left[\int_{q_1^0}^{q_2^0} dq\,\sqrt{E-V_0} - \frac{1}{2}\int_{q_1^0}^{q_2^0}\frac{\epsilon V_1(q)}{\sqrt{E-V_0}}\,dq\right]$$

$$\Rightarrow T = T_0 - \epsilon\sqrt{2m}\,\frac{d}{dE}\int_{q_1^0}^{q_2^0}\frac{V_1(q)}{\sqrt{E-V_0(q)}}\,dq$$

If. $T = T_0(E) + \epsilon T_1(E)$. Then

$$T_1(E) = -\sqrt{2m}\,\frac{d}{dE}\int_{q_1^0}^{q_2^0}\frac{V_1(q)}{\sqrt{E-V_0(q)}}\,dq$$