HOMEWORK 2

Due: Monday, April 11, 2016

1. Try to create an alternative theory to the expanding universe theory that still explains why distance galaxies have redshifted spectra! Suppose that light just loses energy for some unknown reason as it travels through space! Thus suppose that the experimentally measured frequency shift is not due to galaxies moving away, but to $E = h\nu$ decreasing. To make your theory quantitative suppose light loses an amount of energy A for each Mpc it travels, so the change in energy of a photon is

$$\frac{dE}{dr} = -AE.$$

a. Solve the above equation to find the relation between redshift and distance in your new theory.
b. Use a Taylor expansion to show in the limit of small r your theory gives a linear relation just as Hubble found.
c. Use the measured value $H_0 = 70$ km/s/Mpc to find the value of A that makes your theory fit current data.
d. Use your equation to find how far away are galaxies with $z = .02$, $z = 0.2$, $z = 2$, $z = 10$, and $z = 1100$. Are these distances the same, larger, or smaller than found from the expanding Universe theory? (no need to calculate distances with large z in FRW case)

2. Use the non-expanding flat 3-D flat metric $ds^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$ to calculate:
a. the surface area of a sphere with *physical* radius L.
b. the volume of a sphere with physical radius L.

3. Use the non-expanding positively curved 2-D flat metric $ds^2 = dr^2 + R_0^2 \sin^2(r/R_0) d\theta^2$ to calculate:
a. the circumference of a circle of physical radius L. [This is the metric of the surface of a sphere of radius R_0.] [Note physical radius means proper distance, not the coordinate radius; for flat space above this distinction didn't matter, but in this case it is different because the coordinate dr moves along the surface of the sphere, and we measure the radius L along the surface of the sphere.
b. Check your formula by evaluating it for $L = R_0\pi/2$ where it should be a maximum, and $L = R_0\pi$, where you have moved all the way to the other opposite side of the sphere and thus the circumference of the circle should be zero.
c. find the area of a circle of physical radius L. Evaluate and check your formula at $L = R_0\pi/2$ where it should give half the surface area of the sphere, and at $L = R_0\pi$, where it should give $4\pi R_0^2$.
d. Taylor expand your two formulas above for the limit $L \ll R_0$. Show that you get the answers you expect for flat space. (Like the the surface of the Earth seems flat to us.)

4. The Hubble time $t_H = 1/H_0$ sets the scale for the age of the Universe. Using $H_0 = 70$ km/s/Mpc, calculate t_H in years.

5. We defined the Hubble parameter as $H = \dot{a}/a$, where $\dot{a} = da/dt$, and the Hubble "constant" H_0 is just H with a and $\dot{a}$ evaluated today (at $t = t_0$).
(a) Suppose the Hubble parameter were constant (it isn't!). Solve the differential equation $H_0 = \dot{a}/a$, to find a as a function of t. [Hint: integrate from $t = t_1$, $a = a_1$ until a time $t = t$ and $a = a$.]
(b) Now let H change with time and assume $a = (t/t_0)^{2/3}$ as it is in a purely "dust" Universe. For $t_0 = 13.8$ Gyr, use the definition of H to find the Hubble constant H_0 today if this were all that happened. (Give answer both in inverse Gyr and km/s/Mpc.)
(c) In case (b), how many years after the big bang ($t = 0$) was the Universe 1100 times smaller than today?