1. The measured cosmological constant is roughly $\Omega_\Lambda = 0.7$. This means an overall average density of the dark energy in the Universe is $0.7\rho_{crit}$. Using $H_0 = 70km/s/Mpc$ calculate the average density of dark energy in kg per cubic meter. Dark energy is probably spread uniformly over the Universe, so use this number to calculate the total mass of dark energy inside the solar system (say out to Pluto's orbit) (look up the distance to Pluto). Compare this to the mass of the Sun. Will dark energy have much effect on the orbits of the planets?

2. A photon (particle of light) of energy 3.0 GeV and a proton also of total energy 3.0 GeV leave a distant quasar and after a long time reach Earth. The quasar is at a redshift $z = 3$.
(a) What was the scale factor of the Universe when the photon and proton left?
(b) What was the velocity and momentum of the photon and proton when they left? (I've asked for 4 separate numbers here.)(Hint: you need to use formulas from special relativity for the proton, which has a mass of .938 GeV/c^2.)
(c) What is the velocity and momentum and energy of the photon and proton when they arrive on Earth? (I've asked for 6 numbers here.)

3. For $\Omega_m = 1.5$, $\Lambda = 0$, and $h = 0.7$, what is the "radius of the Universe", R_0 (in light years or Mpc)? Remembering how the scale factor works, what was R at redshift $z = 2$? For what value of z was $R = 1$ meter? [Hint: if you have a problem with units remember we sometimes set $c = 1$. You might have to add back in factors of $c = 3 \times 10^5$ km/s $= 1$ lightyear/year, to get the units to work out to meters or lightyears.]

4a. Using the equation from class

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \left[\Omega_m/a^3 + \Omega_{rad}/a^4 + \Omega_V + (1 - \Omega_{tot})/a^2\right]$$

derive the useful equation

$$\frac{dz}{dt} = -H_0(1+z)^2 \left[\Omega_m(1+z) + \Omega_{rad}(1+z)^2 + \Omega_V(1+z)^{-2} + 1 - \Omega_{tot}\right]^{1/2}.$$

b. How long (in years) does it take light to go from a galaxy at $z = 7$ to $z = 6$? For this problem please assume a matter dominated Universe with $\Omega_m = 1$ (you can set Ω_{rad} and Ω_V to zero.)

5. Take $h = 0.7$, $\Omega_{rad} = 0$, $\Omega_m = 0.3$, and $\Omega_V = 0.7$.
(a) Find a formula for how the Hubble "constant" $H(t) = \dot{a}/a$ changes with redshift.
(b) How fast was the universe expanding at $z = 0.1$, $z = 0.4$, $z = 0.7$, $z = 2$, $z = 5$, and $z = 1100$? (in units onf km/s/Mpc)?
(c) Suppose instead that there is the same amount of matter but no vacuum energy. What would the expansion rates be at the above redshifts?
(d) Calculate the percentage differences between the expansion rates for the above redshifts. If you wanted to distinguish between these two possible Universes, which redshifts are most useful to measure?