

1. Consider a type of “vacuum” or dark energy, sometimes called Quintessence which is currently being investigated. This material has an equation of state :  $p = w\rho$ , where  $p$  is pressure,  $\rho$  is energy density, and  $w$  is a constant between -1 and 1 ( $-1 \leq w \leq 1$ ). It has been hypothesized that this material could exist today in some quantum field theories as a time varying cosmological “constant”.

(a) For this material with a flat Universe with  $\Omega_Q = 1$ , find  $\rho$  as a function of scale factor  $a$ . [Hint: Bianchi identity]

(b) Find  $a$  as a function of time  $t$  [Hint: FRW equation before defining  $\Omega$ s].

(c) Show that you get the known answers:  $\rho \propto a^{-3}$ ,  $a \propto t^{2/3}$  for matter (non-relativistic material,  $w = 0$ ),  $\rho \propto a^{-4}$ ,  $a \propto t^{1/2}$  for radiation (relativistic material,  $w = 1/3$ ), and that  $a$  expands exponentially for constant vacuum material ( $w = -1$ ) (Note you will have to back to the basic equation for this last point). Is this material cool or what? How to measure the value of  $w$  is a very hot topic and the goal of many current and planned experiments.

2. Show that for a flat matter dominated Universe, the angular diameter of an extended object is a minimum when its redshift is  $z = 1.25$ . [Hint: start with formula from class, and use co-moving distance in this Universe:  $r = (2c/H_0)/(1 - (1+z)^{-1/2})$ .]

3. For many decades cosmology was thought to be “The search for two numbers”. These were the Hubble constant,  $H_0$ , which we now have measured, and the deceleration parameter,  $q_0$ . Today instead of  $q_0$ , we now measure all the  $\Omega_m$ ,  $\Omega_V$ , etc., but to make contact with all the earlier work it is good to think about  $q_0 \equiv -(\frac{\ddot{a}}{aH^2}) = -(\frac{\ddot{a}a}{\dot{a}^2})$  evaluated today at  $t = t_0$ .

Using the formulas for  $a(t)$  derived from the FRW equation from class and the formula above, evaluate  $q_0$  for the following cases:

(a) Pure matter, flat Universe ( $\Omega_m = \Omega_{tot} = 1$ ,  $\Omega_V = \Omega_{rad} = 0$ ).

(b) Pure radiation, flat Universe ( $\Omega_{rad} = \Omega_{tot} = 1$ ,  $\Omega_V = \Omega_m = 0$ ).

(c) Pure vacuum energy, ( $\Omega_V = \Omega_{tot} = 1$ ,  $\Omega_m = \Omega_{rad} = 0$ ).

4. Consider a QSO with a redshift of  $z = 8.0$ . Assume a flat Universe with  $\Omega_V = 0.7$ , and  $\Omega_m = 0.3$ .

(a) How old was the Universe at this redshift? How long has the light been traveling? Express the results in years. (Hint: Use section 6.2 of Ryden and think about the present age of Universe in this cosmology?)

(b) Write the integral that one should do to find the proper distance today to that QSO. I ran my computer program on this integral and found  $r = 8,950$  Mpc.

(c) Use the above proper distance to find the proper distance (in Mpc) to this QSO when its light was emitted?

(d) What is the luminosity distance to this QSO?

(e) Suppose the angular diameter of the nucleus of the QSO is about 1 arcsec. What is the linear diameter of this object (in units of kpc)?