

1. Write a paragraph explaining where the elements Helium and Deuterium come from. (Be sure your paragraph is grammatically correct and does not contain spelling or factual errors. Also make sure it is 100% in your own words.)
2. Write a paragraph explaining to a fellow physics student the idea of cosmic inflation. Include why we want it, how it works, and why it helps.
3. This problem takes you through some calculations about cosmic inflation theory. Suppose, due to quantum fluctuations, a tiny patch of the Universe the size of a proton (10^{-15}m) suddenly got an inflaton field strength of $\phi = 10^{21}$ GeV. Suppose the inflaton potential is $V = \frac{\lambda}{(\hbar c)^3} \phi^4$, with $\lambda = 10^{-12}$, and $\hbar c = 1.97 \times 10^{-5}$ eV cm.) This small patch begins cosmic inflation as the ϕ -field slowly rolls down the potential towards $\phi = 0$.
 - (a) What is the vacuum energy density ($\rho_V \approx V$) at the beginning of the inflation? (give answer in GeV/cm^3).
 - (b) Suppose there are 60 e-foldings of inflation before it ends. ($a_{end} = a_{begin} \exp(Ht_{inf})$, where $Ht_{inf} = 60$). How many times larger did the patch become during inflation? What is the size of this patch after inflation?
 - (c) Using the FRW equation that relates density to the Hubble parameter find the value of $H = \dot{a}/a$ at the start of inflation. (give answer in inverse seconds (e.g. $H_0 = 1/14$ Gyr). In this equation you may ignore curvature, radiation, and matter.)
 - (d) Using part (c) how long (in seconds) did the inflationary period last?
 - (e) Suppose there were one billion stable particles per cubic centimeter at the beginning of inflation (call them magnetic monopoles). What was their density at the end of inflation? How many monopoles were inside the patch at the end of inflation?
 - (f) Suppose there was some radiation at the beginning of inflation, with a temperature of 10^9GeV . What was the temperature at the end of inflation (Give T in GeV and in K)? Do you see how no matter what is in the Universe when inflation starts, the Universe becomes cold, empty, and flat by the end of inflation? (but before reheating)
 - (g) At the end of inflation suppose the Universe “reheats” by converting all the vacuum energy into radiation. Using the equation $\rho_{rad} = g_* \pi^2 T^4 / (30(\hbar c)^3)$, with $g_* = 107$, calculate the temperature of the Universe after reheating.
 - (h) Suppose this radiation dominated Universe now expands and cools until today. Knowing the temperature of radiation today, how much more does the Universe expand from the end of inflation until today? (you may ignore g_*)
 - (i) What is the size of that original tiny patch today. Give answer in meters and Mpc.
 - (j) How does the size above compare to the current observable Universe (current particle horizon size)? Explain why this means that areas on opposite sides of the sky can be at the same temperature, even though they APPEAR to be out of causal contact. (Problem 1 from HW6).