1. (a) Using the fact that galactic rotation curves are flat, find the density distribtuion of the dark matter. Start with a guess of $\rho_{DM} = Ar^{-n}$, with A and n constants. Do the 3 dimensional integral to calculate the mass, M, within a radius R, $M = \int_0^R \rho d^3r$. Then assume circular motion with Newton's laws to find the velocity as a function of R. Find a value of n which makes v constant.
(b) Using the above, find the value (and units) of the constant A for the Milky Way galaxy. Assume the Milky Way has $v_{rot} = 220$ km/s.
(c) Using your derived formula for $\rho(R)$, find the density of dark matter near the Sun (R=8kpc). Give answer in kg per cubic meter and also in GeV per cubic cm. (We are ignoring the baryons here, which is not really right.)
(d) Using your formula for the galactic mass find the mass of the Milky Way Galaxy out to 50,000pc. Give answers in kg and $M_\odot$. The values you calculated for the dark matter density near the Sun and for the total mass of the Milky Galaxy are not far from the best accepted answers!

2. People estimate the density of dark matter near the Sun as 0.3 GeV/cm^3.
(a) If the dark matter is particles of mass 50 GeV, how many of these dark matter particles are in each cubic meter here in this room?
(b) Suppose instead the mass of the dark matter particle was 15 keV. What is the number density then?
(c) Suppose the dark matter is made entirely of primordial black holes (PBH) of mass $10^{-6} M_\odot$. Estimate the number density of these in this case, and also estimate how far away the nearest PBH is.
(d) If the dark matter is PBHs of mass $1 M_\odot$, how far is the nearest one?

3. Einstein's gravitational light bending equation is:

$$\alpha = \frac{4GM}{c^2 b},$$

where α is the bending angle (in radians), M is the mass of the object, and b is the impact parameter (distance of closet approach).
(a) If a light ray just grazes the limb (edge) of the Sun, what is the light bending angle (in arcseconds). This was Einstein's famous prediction.
(b) If light just grazed the edge of a neutron star, what would the light bending angle be?
(c) If light just grazed the Moon, what would the light bending angle be?