Show all steps in your calculations. Justify all answers. Write clearly.

$hc = 12{,}400 eVA,\ k_B = 1/11{,}600 eV/K,\ m_e c^2 = 511{,}000 eV,\ \mu_B = 5.79\times 10^{-5} eV/T$

$ke^2 = 14.4 eVA,\ \hbar c = 1973\ eVA,\ m_p c^2 = 938.28 MeV,\ \hbar^2/(2m_e) = 3.81 eVA^2$

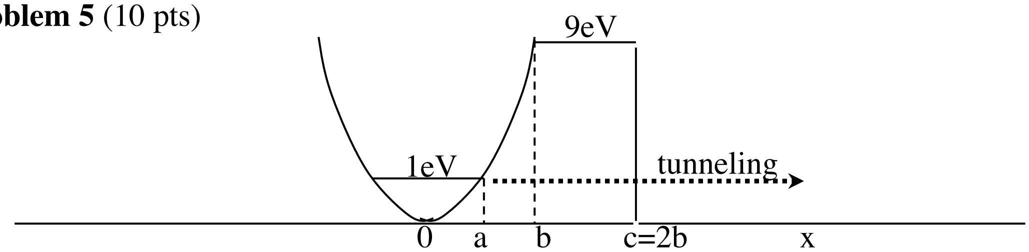
Problem 1 (10 pts+5 extra credit)
Gold (Au) has density ρ=19.3 gr/cm^3, Einstein temperature T_E=100K and atomic weight 197.
A sphere of Au of radius 1cm is initially at 1300K in outer space, far from any stars. Assume it emits radiation as a black body.
(a) How much energy (in J) does it radiate per second?
(b) How long will it take for the temperature of this sphere to drop to 1295K? Give your answer in seconds.
(c) Once its temperature has dropped to 20K, how long will it take to drop 1K more, to 19K? Give your answer in hours.
(d) If the sphere was made of lead (Pb) instead of gold, also approximately a black body, would the answers to (a), (b) or (c) change? If yes, how? (qualitatively only). (Assume Pb is solid at 1300K, even though in reality it melts around 600K).
Hint: the gas constant is R=8.31 J/K mol.

Problem 2 (10 pts)
A gas of He atoms is at temperature 100K and atmospheric pressure.
(a) What is the average kinetic energy of an atom, in eV?
(b) If you pick an atom at random, what is its kinetic energy most likely to be, in eV?
(c) For every 1000 atoms with energy the value found in (b), how many have twice that energy, and how many have 1/2 that energy?

Problem 3 (10 pts)
A photon of wavelength 3.1A is incident on a He^+ ion in its ground state.
(a) If the photon is absorbed, what is the kinetic energy of the electron in the final state? Give your answer in eV.
(b) If the photon is scattered by the electron rather than absorbed, what is the maximum kinetic energy of the electron in the final state?
(c) If the photon is scattered at angle θ with respect to the incident direction, for what range of θ`s (in degrees) will the He^+ ion lose its electron?
(d) If the incident photon is scattered by the He^+ ion and subsequently the ion emits a photon, what are possible values of the wavelength of the emitted photon, and at what angle θ was the incident photon scattered in this case? Give 1 example.

Problem 4 (10 pts)
For the quantum harmonic oscillator the average kinetic and potential energies are the same in any energy state. Using that fact, find $\Delta x \Delta p$ for the first excited state of a quantum harmonic oscillator. Discuss whether or not your answer agrees with Heisenberg's uncertainty principle.

Problem 5 (10 pts)



An electron is in the ground state of the harmonic oscillator + step potential shown in the figure. Its ground state energy is 1eV, assume it is the same as for a regular harmonic oscillator potential.
(a) Find the values of a and b in the figure, in Angstrom. a is the classical amplitude of oscillation in the ground state, b is the value of x for which the harmonic oscillator potential has the value 9eV.
(b) Estimate the tunneling probability through this barrier for the electron in the ground state. For the region $a<x<b$ you can approximate the potential to be constant half-way between 1eV and 9eV.
(c) Estimate how long (in seconds) will it take for this electron to tunnel through the barrier and become unbound. Use that $\hbar = 6.58\times 10^{-16}\, eVs$.

Problem 6 (10 pts)
An electron in a finite square well of height 11.43eV is in the third lowest state of this well and has energy 3.81 eV.
(a) What is the width of this well, in A?
(b) How much more likely is it to find it at the center of the well than at the right boundary of the well?
(c) Make a qualitative plot of the wavefunction versus x both inside and outside the x-range of the well.

Problem 7 (10 pts+3 extra credit)
Hydrogen atoms are at temperature 300K. All atoms have the electron in the n=2 level. There is a magnetic field applied of magnitude 100T. Ignore spin-orbit coupling but do not ignore spin. For every 1000 electrons that have quantum number $\ell = 0$, m_s=-1/2:
(a) How many have $\ell = 0$, m_s=+1/2?
(b) How many have $\ell = 1,\ m = 0,\ m_s = 1/2$?
(c) How many have $\ell = 1,\ m = 1,\ m_s = 1/2$?
(d) For extra credit: how many have $\ell = 1$?

Problem 8 (10 pts)
The distance between atoms in the O_2 molecule is 1.2A. The atomic weight of oxygen is 16.
(a) Find the separation between the lowest and next-lowest rotational energy state of this molecule, in eV, and the characteristic temperature for rotation, T_R, in K.
(atomic mass unit = 931.5 Mev/c^2)
(b) Find the temperature range for which there are more molecules with rotational quantum number $\ell = 3$ than there are for $\ell = 2$ and for $\ell = 4$.