

**Show all steps in your calculations. Justify all answers. Write clearly.**

**Some constants:**  $hc = 12,400 eV\lambda$ ,  $k_B = 1/11,600 eV/K$ ,  $m_e c^2 = 511,000 eV$   
 $e^2 = 14.4 eV\lambda$ ,  $\hbar c = \text{year the prof learned it } eV\lambda$

**Problem 1** (10 pts + 2 extra credit)

In a Rutherford scattering experiment using a foil of atoms of atomic number  $Z$ , Rutherford's  $1/\sin^4(\vartheta/2)$  law is found to hold for all angles between 0 and 150 degrees, and to fail for larger angles.

- For angles larger than 150 degrees, does the experiment find more or fewer scattered particles than predicted by Rutherford's law? Explain.
- In this experiment, the impact parameter for  $\alpha$  particles scattered from a nucleus at a 120 degree angle is  $2.5 \times 10^{-5} \text{A}$ . What can you say about the radius of this nucleus? Give the best possible lower or upper bounds, or both.
- The kinetic energy of the  $\alpha$  particles used in the above experiment was 7.65 MeV. What was  $Z$ , the atomic number of the atoms?
- When the kinetic energy of the  $\alpha$  particles was reduced to 7.25 MeV, Rutherford's  $1/\sin^4(\vartheta/2)$  law was found to hold for all angles. What more can you say about the radius of this nucleus using this information?

**Problem 2** (10 pts)

The electron in a  $\text{He}^+$  ion ( $Z=2$ ) is in a state where it orbits at the same speed as an electron in the ground state of the H atom.

- Find the wavelength of the photon that would be emitted if this electron made a transition to the ground state of the  $\text{He}^+$  ion.
- Find the wavelength of the longest wavelength photon that this electron would absorb in making a transition from this state to another state.
- If this electron obeyed the classical Maxwell laws of electromagnetic radiation, what would be the wavelength of light emitted due to its accelerated motion in its circular orbit? Compare the answer with the average of the wavelengths found in (a) and (b) and comment on its similarity or difference.

Hints: use angular momentum quantization. Use that  $\hbar c / ke^2 = 1/\alpha = 137$ .

**Problem 3** (10 pts)

An electron is confined in a region of linear dimension  $10^{-2} \text{A}$ .

- Using the uncertainty principle estimate the kinetic energy of this electron.
- If you assume that this electron is in the ground state of a hydrogen-like ion and has the kinetic energy calculated in (a), what is the atomic number of this ion?
- What is the radius of the lowest orbit of this ion, and how does it compare to  $10^{-2} \text{A}$ ? Explain why it is larger, smaller or equal.