PHYSICS 200B : CLASSICAL MECHANICS
PROBLEM SET #2

[1] Consider the nonlinear oscillator described by the Hamiltonian

$$H(q,p) = \frac{p^2}{2m} + \tfrac{1}{2}kq^2 + \tfrac{1}{6}\epsilon b q^6 \quad ,$$

where ε is small.

(a) Find the perturbed frequencies $\nu(J)$ to lowest nontrivial order in ϵ.

(b) Find the perturbed frequencies $\nu(A)$ to lowest nontrivial order in ϵ, where A is the amplitude of the q motion.

(c) Find the relationships $\phi = \phi(\phi_0, J_0)$ and $J = J(\phi_0, J_0)$ to lowest nontrivial order in ϵ.

[2] Consider the Hamiltonian

$$H(q,p) = \left(1 + \epsilon\,\frac{q^2}{a^2}\right)\frac{p^2}{2m} + \tfrac{1}{2}m\omega_0^2 q^2 \quad ,$$

where ε is small.

(a) Find the perturbed frequencies $\nu(J)$ to lowest nontrivial order in ϵ.

(b) Find the perturbed frequencies $\nu(A)$ to lowest nontrivial order in ϵ, where A is the amplitude of the q motion.

(c) Find the relationships $\phi = \phi(\phi_0, J_0)$ and $J = J(\phi_0, J_0)$ to lowest nontrivial order in ϵ.

[3] Consider the $n = 2$ Hamiltonian $H(\boldsymbol{J}, \boldsymbol{\phi}) = H_0(\boldsymbol{J}) + \epsilon\, H_1(\boldsymbol{\phi})$, where

$$H_0(\boldsymbol{J}) = \Lambda J_1^{3/2} + \Omega J_2$$
$$H_1(\boldsymbol{\phi}) = \cos\phi_1 \sum_{-\infty}^{\infty} V_n\, e^{in\phi_2} \quad .$$

(a) Obtain an expression for $J_1(t)$ valid to first order in ϵ.

(b) Which tori are destroyed by the perturbation?

[4] Is the following four-dimensional map canonical?

$$x_{n+1} = 2\alpha x_n - \gamma x_n^2 - p_n + X_n^2$$
$$p_{n+1} = x_n$$
$$X_{n+1} = 2\beta X_n - P_n + 2x_n X_n$$
$$P_{n+1} = X_n \quad .$$