PHYSICS 200B : CLASSICAL MECHANICS
PROBLEM SET #3

(1) Create your own pixelated image to iterate under the cat map. You can also find many interesting images over the web. (Nothing pornographic, please![1]) Iterate the pixel coordinates under the cat map. Show how your image gets scrambled after a few iterations of the map, but is nevertheless recurrent. You'll need to write a computer code to do this problem.

(2) Numerically integrate the system

$$\dot{r} = r(1 - r^2) + \lambda r \cos\theta$$
$$\dot{\theta} = 1$$

with $0 < \lambda < 1$, and show that any initial condition lying between the concentric circles of radii $\sqrt{1 \pm \lambda}$ approaches a closed limit cycle in the long time limit. Choose whatever value of λ suits your taste.

(3) Consider the equation

$$\ddot{x} + x = \epsilon\, x^5$$

with $\epsilon \ll 1$.

(a) Develop a two term straightforward expansion for the solution and discuss its uniformity.

(b) Using the Poincaré-Lindstedt method, find a uniformly valid expansion to first order.

(c) Using the multiple time scale method, find a uniformly valid expansion to first order.

(4) Consider the equation

$$\ddot{x} + \epsilon\, \dot{x}^3 + x = 0$$

with $\epsilon \ll 1$. Using the multiple time scale method, find a uniformly valid expansion to first order.

(5) Analyze the forced oscillator

$$\ddot{x} + x = \epsilon \left(\dot{x} - \tfrac{1}{3}\dot{x}^3\right) + \epsilon\, f_0\, \cos(t + \epsilon\nu t)$$

using the discussion in §3.3.1 and §3.3.2 of the notes as a template.

[1] Well, I suppose animal sex is OK, if you must.