Assignment I.

problems for probability concepts

due: October 22, 2017

Code in Matlab or Python

problem 1 PHYS 139/239

You have three coins and you toss all three. When all three landed you count the number of heads and tails.

The first coin is biased with 40 percent probability of heads.
The second coin is biased with 70 percent probability of heads.
The third coin is biased with 65 percent probability of tails.

(a) What is the probability that all three landed on heads?

(b) What is the probability that all three landed on tails?

(c) What is the probability that two are heads and one is tail?

(d) What is the probability that two are tails and one is head?

problem 2 PHYS 139/239

Write computer code which simulates the events of problem 1.

(a) Show in the computer generated event count that it agrees with your theoretical predictions of the four different type of events of problem 1.

(b) How are the counts deviate less and less from the theoretical expectations as you increase the number of simulated events?

(c) Try to give a formula for the estimated deviations.

problem 3 PHYS 139/239

You are an oracle and when asked yes or no questions you do not know the true answer but with probability P = 1/4 you say "Yes" and with probability 1-P = 3/4 you say "No".

Give your theoretically sound procedure for the following problems:

(a) How do you come up with the answer if you only have a fair, two-sided coin which you can toss several times before you give the answer?

(b) Same problem with P = 1/3.

(c) Same problem with P = $1/\pi$ PHYS 239 only

problem 4 PHYS 139/239

Write computer code which simulates the theoretical strategy you developed for solving the questions in problem 3 (now the oracle replaced the coin tossing with the computer).

(a) How does the code give the answer to the oracle with $P = 1/4$?

(b) Same question with $P = 1/3$.

(c) Same question with $P = 1/\pi$ PHYS 239 only

problem 5 PHYS 139/239

(a) Simulate draws from two 7-sided dice whose faces show 1:2:3:5:7:8:9 and the sides are equally probable. What is the probability that the sum of the two dice equals 8?

(b) Simulate draws from two weird 7-sided dice whose faces have probabilities proportional to 1:e:π:4:5:6:e^{π} respectively. What is the probability that the sum of the two dice equals 8? PHYS 239 only

problem 6 PHYS 139/239

Example: The Monty Hall or Let's Make a Deal Problem

- Three doors
- Car (prize) behind one door
- You pick a door, but don't open it yet
- Monty then opens one of the other doors, always revealing no car (he knows where it is)
- You now get to switch doors if you want
- Should you?
- Most people reason: Two remaining doors were equiprobable before, and nothing has changed. So doesn't matter whether you switch or not.

Work out the case for 5 doors when Monty opens two doors

PHYS 239 only: generalize the problem to N doors where Monty opens n doors.

problem 7 PHYS 139/239

Simulate the Knight/Troll/Gnome bridge crossing problem repeatedly in your computer code.

The code counts the number of safe crossings of the Knight when only gnomes are under the bridge. Show that the fraction of counts for safe crossings agrees with the theoretically expected number.

problem 8 PHYS 139/239

Prove that the 3 Kolmogorov axioms are not compatible with assigning non-zero probability to null-event