HW set 4

Problem 1: AM, 6.1

Problem 2
Do one of the following two alternatives, I or II:
I
AM, 8.1
If you don't succeed (as I haven't), do this instead:
II
For a Kronig-Penney model with potential
$U(x) = aU_0\delta(x - na)$
Assume the wavefunction for wavevector k is of the form:
$\psi_k(x) = Ae^{iqx} + Be^{-iqx}$
in the interval
$0 \le x \le a$
and that it satisfies the Bloch condition
$\psi_k(x + na) = e^{ikna}\psi_k(x)$
for any integer n
(a) For a given q, what is the band energy ε_k ?
(b) Find an equation for A and B by requiring that the wavefunction is continuous at x=0.
(c) Find a second equation for A and B by integrating the Schrodinger eq. for this wavefunction between $-\delta$ and δ with $\delta \to 0$ (i.e. δ very small).
(d) By setting the determinant = 0, find an equation relating k and q.
Hint: the equation looks like
$\cos(ka) = \cos(qa) + C\sin(qa)$
You need to find C.
(e) Make a graph of the energy versus k relation for the three lowest energy bands, for
$U_0 \frac{ma^2}{\hbar^2} = \frac{3}{2}\pi$
(f) Same as (e) for
$U_0 \frac{ma^2}{\hbar^2} = \frac{3}{4}\pi$