PHYSICS 152B/232
Spring 2017
Homework Assignment #3

[1] Consider a material with one orbital per site on a simple cubic lattice. The nearest neighbor hopping integral is $t = 1.0\,\mathrm{eV}$ and the lattice spacing is $a = 5\,\text{Å}$. The system is doped so that the Fermi level lies at $\varepsilon_{\rm F} = -2.0\,\mathrm{eV}$. Compute the period of de Haas-van Alphen oscillations $\Delta\big(\frac{1}{B}\big)$ when the field is in the $\hat{\boldsymbol{z}}$ direction.

[2] *Cyclotron resonance in Si and Ge* – Both Si and Ge are indirect gap semiconductors with anisotropic conduction band minima and doubly degenerate valence band maxima. In Si, the conduction band minima occur along the $\langle 100\rangle$ ($\langle \Gamma\mathrm{X}\rangle$) directions, and are six-fold degenerate. The equal energy surfaces are cigar-shaped, and the effective mass along the $\langle \Gamma\mathrm{X}\rangle$ principal axes (the 'longitudinal' effective mass) is $m_{\rm l}^* \simeq 1.0\,m_{\rm e}$, while the effective mass in the plane perpendicular to this axis (the 'transverse' effective mass) is $m_{\rm t}^* \simeq 0.20\,m_{\rm e}$. The valence band maximum occurs at the unique Γ point, and there are two isotropic hole branches: a 'heavy' hole with $m_{\rm hh}^* \simeq 0.49\,m_{\rm e}$, and a 'light' hole with $m_{\rm lh}^* \simeq 0.16\,m_{\rm e}$.

In Ge, the conduction band minima occur at the fourfold degenerate L point (along the eight $\langle 111\rangle$ directions) with effective masses $m_{\rm l}^* \simeq 1.6\,m_{\rm e}$ and $m_{\rm t}^* \simeq 0.08\,m_{\rm e}$. The valence band maximum again occurs at the Γ point, where the hole masses are $m_{\rm hh}^* \simeq 0.34\,m_{\rm e}$ and $m_{\rm lh}^* \simeq 0.044\,m_{\rm e}$. Use the following figures to interpret the cyclotron resonance data shown below. Verify whether the data corroborate the quoted values of the effective masses in Si and Ge.

(See the following four figures.)

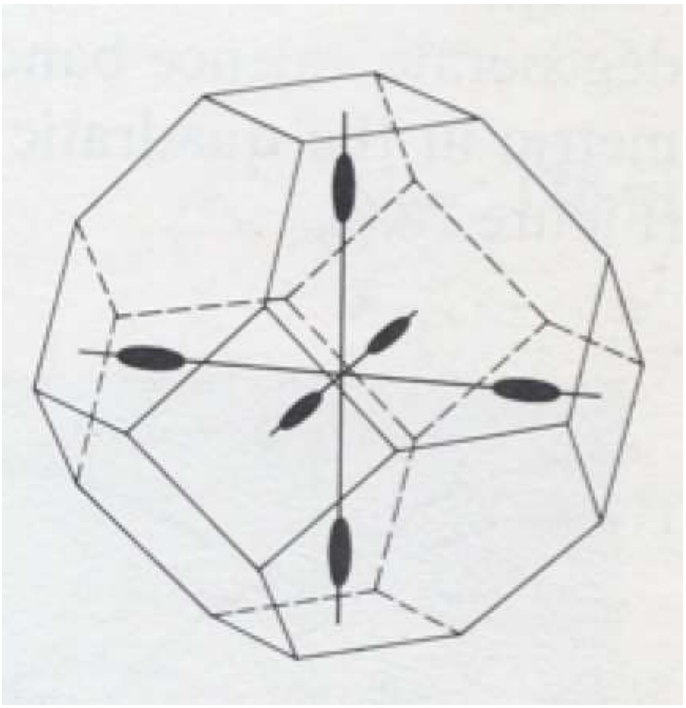

Figure 1: Constant energy surfaces near the conduction band minima in silicon. There are six symmetry-related ellipsoidal pockets whose long axes run along the $\langle 100 \rangle$ directions.

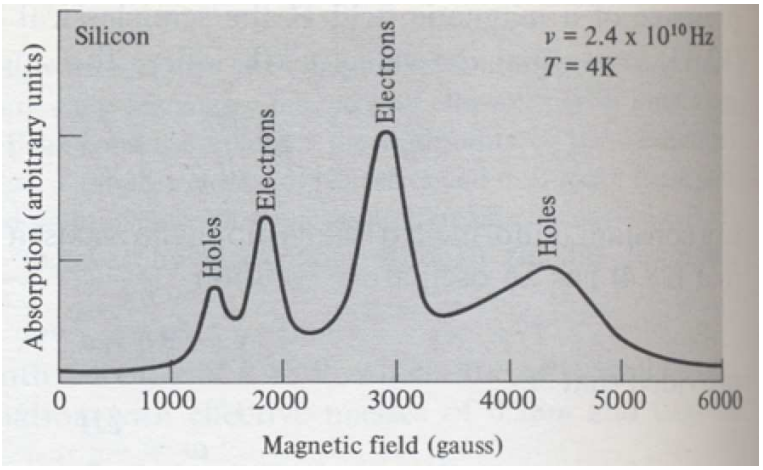

Figure 2: Cyclotron resonance data in Si (G. Dresselhaus *et al.*, *Phys, Rev*, **98**, 368 (1955).) The field lies in a (110) plane and makes an angle of 30° with the [001] axis.

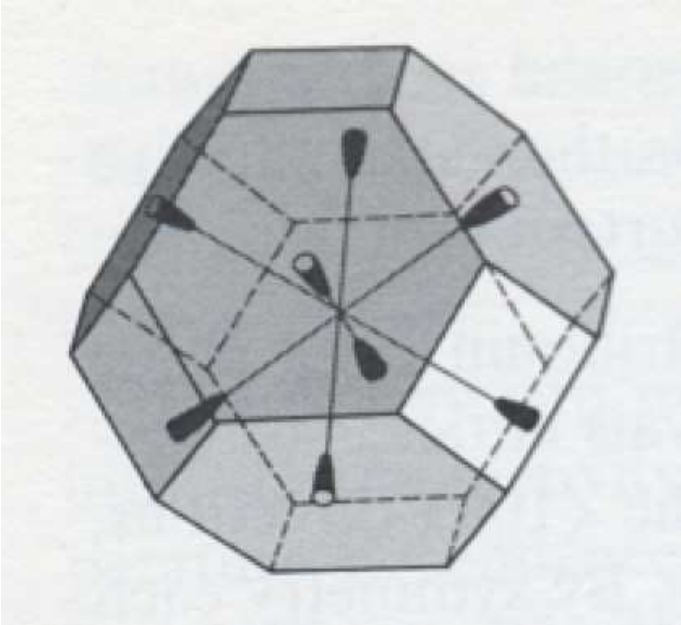

Figure 3: Constant energy surfaces near the conduction band minima in germanium. There are eight symmetry-related half-ellipsoids whose long axes run along the $\langle 111 \rangle$ directions, and are centered on the midpoints of the hexagonal zone faces. With a suitable choice of primitive cell in $\boldsymbol{k}$-space, these can be represented as four ellipsoids, the half-ellipsoids on opposite faces being joined together by translations through suitable reciprocal lattice vectors.

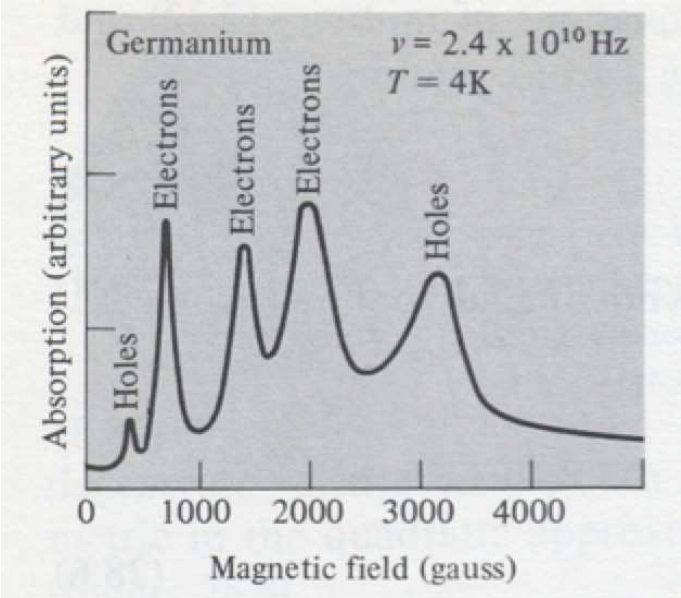

Figure 4: Cyclotron resonance data in Ge (G. Dresselhaus *et al.*, *Phys, Rev,* **98**, 368 (1955).) The field lies in a (110) plane and makes an angle of 60° with the [001] axis.