HW problem week 6

Your turned in assignment should be clearly written and easy to follow! Learning how to explain your work in a way that is as easy as possible to follow is an important part of your training as a physicist. An incoherent mess of equations with a correct final answer could receive less points than a solution which is clearly explained at every step but has an algebra mistake somewhere. Once you've solved the problem, you can rewrite it on a new piece of paper for clarity if you need to.

The normalized eigenfunctions of the infinite square well potential are $\psi_n(x) = \sqrt{\frac{2}{L}}\sin(n\pi x/L)$ inside the well.

1. Use these eigenstates to solve the time dependent schrodinger equation to find $\Psi_n(x,t)$. Using the full time dependent wave function, calculate $\langle x\rangle(t)$ and $\langle p\rangle(t)$ in the n'th energy eigenstate.

2. Calculate the average energy $\langle E\rangle(t)$ and the average square energy $\langle E^2\rangle(t)$ to find the uncertainty in the energy $\Delta E = \sqrt{\langle E^2\rangle - \langle E\rangle^2}$. Hint: use the fact that the ψ_n are eigenstates of $\hat{H}$.

3. Now consider the superposition state

   $$\Phi(x,t) = \frac{1}{\sqrt{2}}\left(\Psi_1(x,t) + \Psi_4(x,t)\right).$$

   Verify that this state is normalized.

4. For this new state Φ, calculate $\langle x\rangle(t)$ and $\langle p\rangle(t)$. Would you call the state $\Phi(x,t)$ a stationary state? Why or why not?

5. Repeat part 2 for the state Φ. Hint: be careful in how you apply the hint from part 2.

6. Using the time-energy uncertainty principle $\Delta E\Delta t > \hbar/2$, estimate approximately how much time the particle spends in a particular eigenstate state before flipping to the other one.