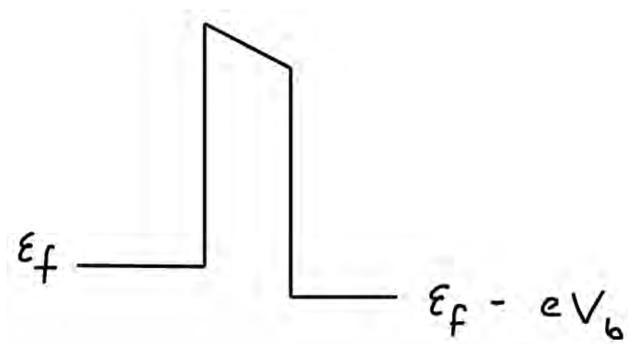
# HW problem week 7

In this problem, we will consider electrons tunnelling between the surface of a metal and a probe which is very close to - but not touching - the surface. In part 5, you will need to do a little bit of research so as always, make sure you cite your sources. You don't need to include a full citation, but you should acknowledge where you found your information. For example, you can say 'I found this data on hyperphysics', or 'data taken from the Hirsch lab website'.

1. Consider two samples of (the same type of) metal $s_1$ and $s_2$ whose surfaces are a distance $d$ apart. Suppose that within each of the samples, the electrons fill energy levels up to an energy $\epsilon_f$. Give a qualitative expression for the transmission coefficient from one sample to the other. Hint: the diagram on page 133 of the textbook may be useful.

2. If we measure the current flowing between the samples, what do you expect to find? Why?

3. Let's now call $s_1$ the STM probe and $s_2$ the metal sample we are measuring. Consider applying a voltage difference $V_b$ between the probe and the surface. This leads to a linear potential energy in between the sample and the probe.




   Show that if the voltage bias is greater than the work function, the electron in effect needs to tunnel through a triangular barrier. What is the 'effective width' of the barrier that the electron sees?

4. By dividing the gap between the surface and the probe into many small intervals and treating each interval as a rectangular barrier of a given height, show that the transmission through this barrier is approximately given by

$$T \sim \exp\left(-2\int_0^{d_{eff}} \sqrt{2m(U(x)-\epsilon_f)}/\hbar\right).$$

Using the appropriate expression for $U(x)$, evaluate the integral to find $T$ for the case $V_b = \phi$.

5. An approximate expression for the number of electrons tunnelling between the probe and surface per second is $nv_fTA$, where $n$ is the number density, $v_f$ is the fermi velocity, and A is the area of the tip. Do some research and find the values of $n, v_f$, and $\phi$ for a metal of your choice, as well as the typical size of the tip of an STM probe. Find an order of magnitude estimate of the tunnelling current when the probe is 1 nm from the surface. Estimate the vertical resolution of the STM by finding the change in $d$ which results in a change in the current by a factor of two. For this problem, assume $V_b \ll \phi$ so that the barrier is approximately rectangular.