HW problem week 7

In this problem, we will consider electrons tunnelling between the surface of a metal and a probe which is very close to - but not touching - the surface. In part 5, you will need to do a little bit of research so as always, make sure you cite your sources. You don't need to include a full citation, but you should acknowledge where you found your information. For example, you can say 'I found this data on hyperphysics', or 'data taken from the Hirsch lab website'.

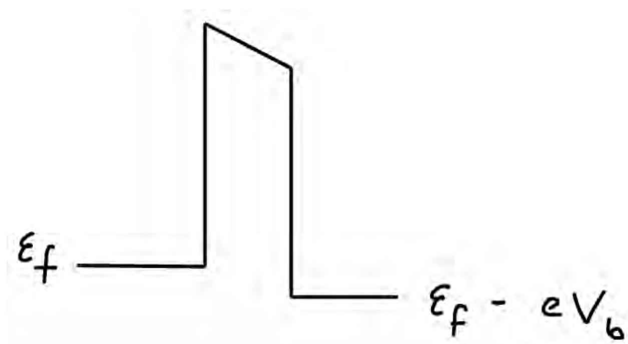
1. Consider two samples of (the same type of) metal s_1 and s_2 whose surfaces are a distance d apart. Suppose that within each of the samples, the electrons fill energy levels up to an energy ϵ_f. Give a qualitative expression for the transmission coefficient from one sample to the other. Hint: the diagram on page 133 of the textbook may be useful.

   The electrons in either sample are treated as plane waves. Only the electrons near the top of the filled electron sea with energy ϵ_f will have a significant probability to tunnel, and the energy barrier for these electrons is ϕ, the work function. The tunnelling probability is therefore proportional to $e^{-2d\sqrt{2m\phi}/\hbar}$.

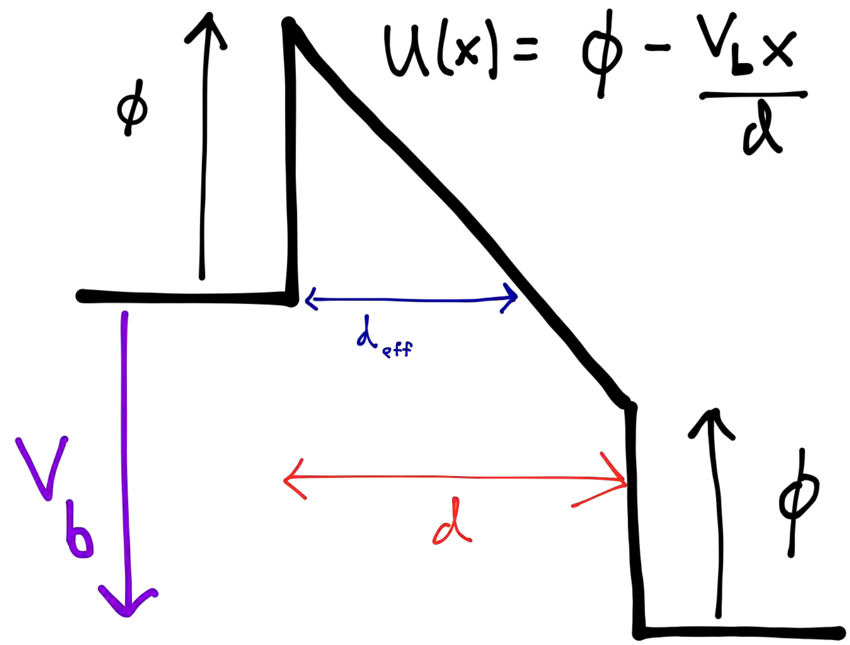
2. If we measure the current flowing between the samples, what do you expect to find? Why?

   Zero, for two reasons. First, both samples have all their electron states filled up to energy ϵ_f, so there are no states available to tunnel into. Second, transmission probabilities are equal in both directions, so there should not be an excess of electrons flowing in any direction.

3. Let's now call s_1 the STM probe and s_2 the metal sample we are measuring. Consider applying a voltage difference V_b between the probe and the surface. This leads to a linear potential energy in between the sample and the probe.



Show that if the voltage bias is greater than the work function, the electron in effect needs to tunnel through a triangular barrier. What is the 'effective width' of the barrier that the electron sees?



When $V_b \geq \phi$, the height of the barrier becomes zero at some smaller distance $d_{eff} \leq d$ and the electron effectively sees a triangular barrier. The effective width is where $\phi - V_b x/d = 0$ or $x = \phi d/V_b$.

4. By dividing the gap between the surface and the probe into many small intervals and treating each interval as a rectangular barrier of a given height, show that the transmission through this barrier is approximately given by

$$T \sim \exp\left(-2\int_0^{d_{eff}} \sqrt{2m(U(x)-\epsilon_f)}/\hbar\right).$$

Using the appropriate expression for $U(x)$, evaluate the integral to find T for the case $V_b = \phi$.

We chop up the region into many small intervals dx, each of which we treat as a rectangular barrier of width dx and height $U(x)$. The transmission through a single rectangular barrier is $\sim e^{-2\kappa(x)dx}$ where $\kappa(x) = \sqrt{2m(U(x)-\epsilon_f)}/\hbar$. To get the overall transmission coefficient through multiple barriers, we multiply the individual transmission coefficients

$$T = \prod_i T_i = \prod_i e^{-2\kappa(x_i)dx} = e^{-2\sum_i \kappa(x_i)dx} \rightsquigarrow e^{-2\int \kappa(x)dx}.$$

Now $U(x)$ is $\epsilon_f + \phi - V_b x/d$ so the argument in the exponential is

$$-2\int_0^d dx\sqrt{2m(\phi - V_b x/d)}/\hbar = -\frac{4\sqrt{2}d\sqrt{m\phi}}{3\hbar} = -\frac{2}{3}\times 2d\sqrt{2m\phi/\hbar^2}$$

5. An approximate expression for the number of electrons tunnelling between the probe and surface per second is $nv_f TA$, where n is the number density, v_f is the fermi velocity, and A is the area of the tip. Do some research and find the values of n, v_f, and ϕ for a metal of your choice, as well as the typical size of the tip of an STM probe. Find an order of magnitude estimate of the tunnelling current when the probe is 1 nm from the surface. Estimate the vertical resolution of the STM by finding the change in d which results in a change in the current by a factor of two. For this problem, assume $V_b \ll \phi$ so that the barrier is more rectangular than triangular.

   Approximate values: $\phi \sim 4\ eV$, $n \sim 0.1/\mathring{A}^3$, $v_f \sim 0.01c$, $A \sim 10^5 \mathring{A}^2$ Using these values, $T \sim 10^{-9}$. The current is charge/sec so we have to multiply by the electron charge as well. Using these numbers gives $I \sim 10^{-8}$, which is ballpark correct.

   If we rewrite the exponential factor as e^{-d/d_0} then d_0 sets the length scale over which the current varies by a factor of e. Here, $d_0 = (2\sqrt{2m\phi/\hbar^2})^{-1} \approx 2\mathring{A}$.