Show all steps in your calculations. Justify all answers. Write clearly.
Suggestion: do the problems you find easiest first

Some constants: $hc = 12{,}400\,eVA,\ \ k_B = 1/11{,}600\,eV/K,\ \ m_ec^2 = 511{,}000\,eV$

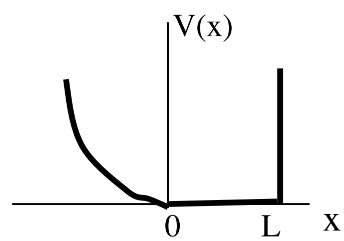
$\hbar c = 1973\,eVA$; $ke^2 = 14.4\,eVA$; $1A = 10^{-10}m$; $m_{neutron}c^2 = 939.6\,MeV$

$\hbar^2/m_e = 7.62\,eVA^2 \quad \mu_B = 5.79\times 10^{-5}\,eV/T$

Problem 1 (10 pts)
A gas of hydrogen-like atoms at room temperature absorbs photons of wavelength 41.03A.
(a) After absorbing such photons, what are all the possible wavelengths of photons it will emit?
(b) At what temperature will this gas have the same number of atoms in the ground state as in the first excited state? Hint: it's very high.

Problem 2 (10 pts)
A black body at temperature T emits maximum power per unit wavelength at wavelength λ=2000A, 1μW of power.
(a) How much power per unit wavelength does it emit at λ=4000A?
(b) How much power per unit wavelength does it emit at λ=1000A?
(c) At what wavelength larger than 2000A does it emit approximately $10^{-4}\mu$W per unit wavelength?

Problem 3 (10 pts)
An electron moves in the potential shown in the figure:



$$V(x) = \frac{1}{2}m\omega^2x^2 \qquad x<0$$
$$V(x) = 0 \qquad 0<x<L$$
$$V(x) = \infty \qquad x>L$$

m is the electron mass, $\hbar\omega = 2eV$, and the electron energy is 1eV.
(a) From the information given, it follows that for x<0 the wavefunction has the form $\psi(x) = e^{-\lambda x^2}$. Explain why that is.
(b) Find the smallest possible value that L can have, in A.
(c) Find the next smallest value that L can have.
(d) Make a plot of the wavefunction of the electron for cases (b) and (c).
Hint: use continuity of the wavefunction and its derivative at x=0.

Problem 4 (10 pts)
There are 10 electrons in a two-dimensional square box of side length L=6A. Electrons have spin 1/2 and obey the Pauli principle, for this problem we assume that they don't interact with each other and that there is no spin-orbit coupling.
(a) Find the ground state energy of this system, in eV.
(b) Find the wavelength of the longest wavelength photon that this system can absorb when it is in the ground state, in A.

Problem 5 (10 pts)
The operator L^2 is given by

$$L^2 = -\hbar^2\left[\frac{\partial^2}{\partial\theta^2} + \frac{\cos\theta}{\sin\theta}\frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]$$

Consider the function $F(\theta,\phi) = (\cos^2\theta + a)e^{i\alpha\phi}$, where a and α are constants.
Find all the possible values of a and α for which $F(\theta,\phi)$ is an eigenfunction of L^2, and the corresponding eigenvalues. Justify your answers by explicit calculation.

Problem 6 (10 pts)
An electron is in the ground state of a spherically symmetric harmonic oscillator potential and has energy 0.06eV. Ignore electron spin.
(a) Find the wavelength of the photon it can absorb (in A).
(b) In the presence of a magnetic field of magnitude 90T, find the wavelengths of all the photons it can absorb.

Problem 7 (10 pts)
The density of states for electrons in a box of volume V is

$$g(E) = CVE^{1/2}$$

where C is a constant.
(a) For N electrons in this box, find an expression for the Fermi energy E_F in terms of C, V and N.
(b) Find an expression for the total energy of this system at zero temperature, E_T, expressed in terms of N and E_F only.
(c) The pressure that the electrons exert is given by

$$P = -\frac{dE_T}{dV}$$

Calculate P and show that for this system, similarly to the classical ideal gas,

$$PV = a\ (nRT_F)$$

where T_F is the Fermi temperature ($T_F = E_F/k_B$), R is the gas constant, n is the number of moles, and a is a number. Find a.

Problem 8 (10 pts)
The He_2^+ molecular ion has bond length 1.08A.
(a) Find its characteristic temperature for rotation T_R (in K), above which the equipartition theorem holds for rotation for a gas of such molecules.
(b) Find the difference in energy (in eV) between two neighboring peaks in the absorption spectrum resulting from transitions between vibrational and rotational energy levels.
(c) Find for which rotational energy level ℓ the thermal equilibrium occupation is largest at room temperature (300K).