

Show all steps in your calculations. Justify all answers. Write clearly.

Some constants: $hc = 12,400\text{eV}\cdot\text{\AA}$, $k_B = 1/11,600\text{eV}/\text{K}$, $m_e c^2 = 511,000\text{eV}$

Problem 1 (10 pts)

In an ideal gas at temperature T the molecules of mass m are confined to move only in two dimensions, x and y , with normalized velocity distribution

$$F(v_x, v_y) = \left(\frac{m}{2\pi k_B T}\right) e^{-m(v_x^2 + v_y^2)/(2k_B T)}$$

(a) Verify that this distribution is normalized, using that $\int_{-\infty}^{\infty} dx e^{-\lambda x^2} = \sqrt{\frac{\pi}{\lambda}}$.

(b) Find the normalized speed distribution $g(v)$, where $g(v)dv$ is the probability that a molecule has speed between v and $v+dv$. Show explicitly that the $g(v)$ that you find is normalized.

(c) The average speed $\langle v \rangle$ is given by $\langle v \rangle = \sqrt{A \frac{k_B T}{m}}$, where A is a number. Find A .

(d) The rms speed is given by $v_{rms} = \sqrt{\langle v^2 \rangle} = \sqrt{B \frac{k_B T}{m}}$, where B is a number. Find B , using either $F(v_x, v_y)$ or $g(v)$.

$$\text{Hints: } \int_0^{\infty} dx x^{n+2} e^{-\lambda x^2} = -\frac{d}{d\lambda} \int_0^{\infty} dx x^n e^{-\lambda x^2}; \quad \int_0^{\infty} dx x e^{-\lambda x^2} = \frac{1}{2\lambda}$$

Problem 2 (10 pts)

A black body at temperature $T=4,674\text{K}$ emits $1\mu\text{W}$ of power in the wavelength range $620,000\text{\AA}$ to $620,001\text{\AA}$.

(a) What is the maximum power that this body emits within a wavelength range of 1\AA ? For what wavelength is that?

(b) At which wavelength (approximately) larger than the one found in (a) does this body emit $16\mu\text{W}$ of power within a wavelength range of 1\AA ?

(c) Find a wavelength that is smaller than the one found in (a) where the body emits less than $1\mu\text{W}$ of power in a wavelength range of 1\AA around that wavelength.

Problem 3 (10 pts)

In a Compton scattering experiment the maximum kinetic energy of the scattered electron is 248eV .

(a) What is the wavelength of the incident photon, in \AA ?

(b) What is the minimum kinetic energy of the scattered electron, in eV ?

(c) When the scattered electron has kinetic energy 100eV , at what angle is the photon scattered? Give your answer in degrees.

(d) For extra credit (3 pts): When the scattered electron has the largest possible momentum in direction perpendicular to the incident direction, at what angle is the photon scattered? Give your answer in degrees.