Physics 4E Quiz 2 Solutions Spring 2017

## Problem 1

(a) $r_d = \dfrac{k q_\alpha Q}{E_\alpha} = \dfrac{k e^2 \cdot 2 \cdot 50}{E_\alpha} = \dfrac{14.4\,\mathrm{eV}\cdot\mathring{A} \times 100}{7.2\times10^{6}\,\mathrm{eV}} = 2\times10^{-4}\,\mathring{A}$

$$\boxed{r_d = 2\times10^{-4}\,\mathring{A}}$$

(b) $b = \dfrac{k q_\alpha Q}{2E_\alpha}\cdot\cot\dfrac{\theta}{2} = \dfrac{r_d}{2}\cdot\cot\dfrac{\theta}{2} = 10^{-4}\mathring{A}$, since $\cot 45° = 1$

(c) If the incident beam covers an area A of the foil and has N $\alpha$-particles, each nucleus will scatter $N\cdot\dfrac{\pi b^2}{A}$ by an angle $> 90°$. The number of scattering nuclei is $n\cdot t\cdot A$, where $n$ = # of nuclei / unit volume and $t$ = thickness of foil. So,

$$N(\theta>90°) = N\cdot\frac{\pi b^2}{A}\times ntA = N\cdot\pi b^2\cdot nt$$

$$nt = \frac{0.333}{\mathring{A}^3}\times 30{,}000\,\mathring{A} = \frac{10{,}000}{\mathring{A}^2} \Rightarrow$$

$$N(\theta>90°) = N\cdot\pi\cdot10^{-8}\mathring{A}^2\times\frac{10^4}{\mathring{A}^2} = 10^7\cdot10^{-4}\pi = \boxed{3{,}142\ \alpha\text{-particles}}$$

(d) $N(90<\theta<92) = N\cdot nt\cdot 2\pi b\cdot db$

$b(\theta=92) = b(\theta=90)\cdot\cot(46°)/\cot(45°) = 0.966 \Rightarrow$

$\Rightarrow db = b(\theta=90) - b(\theta=92) = b(\theta=90)\times 0.034$

$$\Rightarrow N(90<\theta<92) = \underbrace{N\cdot nt\cdot\pi b^2}_{N(\theta>90)}\times 2\times0.034 = \boxed{214\ \alpha\text{-particles}}$$

Problem 2

Frequency of revolution $f = \frac{v}{2\pi r}$ , $f = \frac{c}{\lambda}$ $\Rightarrow$

$\lambda = \frac{2\pi r \cdot c}{v}$ . In Bohr model, $L = mvr = n\hbar \Rightarrow \frac{v}{r} = \frac{n\hbar}{mr^2}$

and $r = a_0 n^2$, so $\lambda_n = 2\pi \frac{mr^2 c}{n\hbar} = \frac{2\pi m c a_0^2 n^3}{\hbar}$

$$\boxed{\lambda_n = \frac{2\pi m c a_0^2 n^3}{\hbar}}$$

(b) $a_0 = 0.529\,Å$, $mc^2 = 511{,}000\,eV$, $\hbar c = 1973\,eV\,Å$ $\Rightarrow$

$\lambda_n = \frac{2\pi \cdot 511{,}000 \times 0.529^2 n^3}{1973} Å$ $\Rightarrow$

$\boxed{\lambda_n = 455.4\,Å \cdot n^3}$ $\Rightarrow$ $\boxed{\lambda_4 = 29{,}145\,Å\ ,\ \lambda_5 = 56{,}924\,Å}$

(c) $hf = \frac{hc}{\lambda_{mn}} = E_0\left(\frac{1}{m^2} - \frac{1}{n^2}\right) = 13.6\,eV \cdot \left(\frac{1}{4^2} - \frac{1}{5^2}\right) = 0.306\,eV$

$\Rightarrow \lambda_{45} = \frac{12{,}400}{0.306} Å = \boxed{40{,}523\,Å}$

and $\frac{\lambda_4 + \lambda_5}{2} = \boxed{43{,}044\,Å}$

they are close, but not equal. They become closer and closer as $n \to \infty$ (correspondence principle).

## Problem 3

$\lambda = \frac{h}{p}$ de Broglie wavelength. Assume $\lambda_n = \lambda_e$

Classically, $E_k = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mE_k}$, so

$$\frac{\lambda_e}{\lambda_n} = \frac{p_n}{p_e} = \sqrt{\frac{m_n E_{k,n}}{m_e E_{k,e}}} \Rightarrow \frac{E_{k,e}}{E_{k,n}} = \frac{m_n}{m_e} \cdot \left(\frac{\lambda_n}{\lambda_e}\right)^2$$

For $\lambda_n = \lambda_e$, $\boxed{\frac{E_{ke}}{E_{kn}} = \frac{m_n}{m_e} = \frac{939.6}{0.511} = 1839}$

(b) extremely relativistic: $pc \gg mc^2$

$$E_k = E - mc^2 = \sqrt{p^2c^2 + m^2c^4} - mc^2 \longrightarrow pc \text{ for extremely relativistic.}$$

So $\boxed{\frac{E_{ke}}{E_{kn}} = \frac{p_e}{p_n} = \frac{\lambda_n}{\lambda_e} = 1}$

(c) Define relativistic regime as having minimum $p$ that satisfies

$$E_k = \sqrt{p^2c^2 + m^2c^4} - mc^2 = mc^2 \Rightarrow$$

$$\Rightarrow p^2c^2 = 3m^2c^4 \Rightarrow p = \sqrt{3}\, mc$$

$$\lambda_n = \frac{h}{\sqrt{3}\, m_n c} = \frac{12{,}400}{\sqrt{3} \times 939.6 \times 10^6} \text{Å} = 7.6 \times 10^{-6} \text{Å}$$

So the range of de Broglie wavelengths for this neutron is

$$\boxed{0 < \lambda_n < 7.6 \times 10^{-6} \text{Å}}$$