1-5 This is a case of dilation. $T = \gamma T'$ in this problem with the proper time $T' = T_0$

$$T = \left[1 - \left(\frac{v}{c}\right)^2\right]^{-1/2} T_0 \Rightarrow \frac{v}{c} = \left[1 - \left(\frac{T_0}{T}\right)^2\right]^{1/2};$$

in this case $T = 2T_0$, $v = \left\{1 - \left[\frac{L_0/2}{L_0}\right]^2\right\}^{1/2} = \left[1 - \left(\frac{1}{4}\right)\right]^{1/2}$ therefore $v = 0.866c$.

1-6 This is a case of length contraction. $L = \frac{L'}{\gamma}$ in this problem the proper length $L' = L_0$,

$L = \left[1 - \frac{v^2}{c^2}\right]^{-1/2} L_0 \Rightarrow v = c\left[1 - \left(\frac{L}{L_0}\right)^2\right]^{1/2}$; in this case $L = \frac{L_0}{2}$,

$v = \left\{1 - \left[\frac{L_0/2}{L_0}\right]^2\right\}^{1/2} = \left[1 - \left(\frac{1}{4}\right)\right]^{1/2}$ therefore $v = 0.866c$.

1-8 $L = \frac{L'}{\gamma}$

$$\frac{1}{\gamma} = \frac{L}{L'} = \left[1 - \frac{v^2}{c^2}\right]^{1/2}$$

$$v = c\left[1 - \left(\frac{L}{L'}\right)^2\right]^{1/2} = c\left[1 - \left(\frac{75}{100}\right)^2\right]^{1/2} = 0.661c$$

1-9 $L_{\text{earth}} = \frac{L'}{\gamma}$

$L_{\text{earth}} = L'\left[1 - \frac{v^2}{c^2}\right]^{1/2}$, L', the proper length so $L_{\text{earth}} = L = L\left[1 - (0.9)^2\right]^{1/2} = 0.436L$.

1-10 (a) $\tau = \gamma \tau'$ where $\beta = \frac{v}{c}$ and

$$\gamma =' \left(1 - \beta^2\right)^{-1/2} = \tau'\left(1 - \frac{v^2}{c^2}\right)^{-1/2} = \left(2.6 \times 10^{-8}\text{ s}\right)\left[1 - (0.95)^2\right]^{-1/2} = 8.33 \times 10^{-8}\text{ s}$$

(b) $d = v\tau = (0.95)\left(3 \times 10^8\right)\left(8.33 \times 10^8\text{ s}\right) = 24\text{ m}$

1-12 (a) 70 beats/min or $\Delta t' = \frac{1}{70}$ min

(b) $\Delta t = \gamma\, \Delta t' = \left[1 - (0.9)^2\right]^{-1/2}\left(\frac{1}{70}\right)$ min $= 0.032\,8$ min/beat or the number of beats per minute $\approx 30.5 \approx 31$.

1-16 For an observer approaching a light source, $\lambda_{\text{ob}} = \left[\frac{(1-v/c)^{1/2}}{(1+v/c)^{1/2}}\right]\lambda_{\text{source}}$. Setting $\beta = \frac{v}{c}$ and after some algebra we find,

$$\beta = \frac{\lambda_{\text{source}}^2 - \lambda_{\text{obs}}^2}{\lambda_{\text{source}}^2 + \lambda_{\text{obs}}^2} = \frac{(650\text{ nm})^2 - (550\text{ nm})^2}{(650\text{ nm})^2 + (550\text{ nm})^2} = 0.166$$

$$v = 0.166c = (4.98\times 10^7\ \text{m/s})(2.237\ \text{mi/h})(\text{m/s})^{-1} = 1.11\times 10^8\ \text{mi/h}.$$

1-20 $u = \frac{v+u'}{1+vu'/c^2} = \frac{0.90c + 0.70c}{1+(0.90c)(0.70c)/c^2} = 0.98c$

1-21 $u'_X = \frac{u_X - v}{1-u_X v/c^2} = \frac{0.50c - 0.80c}{1-(0.50c)(0.80c)/c^2} = -0.50c$

1-23 (a) Let event 1 have coordinates $x_1 = y_1 = z_1 = t_1 = 0$ and event 2 have coordinates $x_2 = 100$ mm, $y_2 = z_2 = t_2 = 0$. In S', $x'_1 = \gamma(x_1 - vt_1) = 0$, $y'_1 = y_1 = 0$, $z'_1 = z_1 = 0$, and $t'_1 = \gamma\left[t_1 - \left(\frac{v}{c^2}\right)x_1\right] = 0$, with $\gamma = \left[1-\frac{v^2}{c^2}\right]^{-1/2}$ and so $\gamma = \left[1-(0.70)^2\right]^{-1/2} = 1.40$. In system S', $x'_2 = \gamma(x_2 - vt_2) = 140$ m, $y'_2 = z'_2 = 0$, and

$$t'_2 = \gamma\left[t_2 - \left(\frac{v}{c^2}\right)x_2\right] = \frac{(1.4)(-0.70)(100\text{ m})}{3.00\times 10^8\ \text{m/s}} = -0.33\ \mu\text{s}.$$

(b) $\Delta x' = x'_2 - x'_1 = 140$ m

(c) Events are not simultaneous in S', event 2 occurs $0.33\ \mu$s earlier than event 1.

1-26 The observed length of an object moving with speed v is $L = L'\left[1-\left(\frac{v}{c}\right)^2\right]^{1/2}$ with L' being the proper length. For the two ships, we know that $L_2 = L_1$, $L'_2 = 3L'_1$ and $v_1 = 0.35c$. Thus $L_2^2 = L_1^2$ and $(9L_1'^2)\left[1-\left(\frac{v_2}{c}\right)^2\right] = L_1'^2\left[1-(0.35)^2\right]$, giving $9 - 9\left(\frac{v_2}{c}\right)^2 = 0.877\ 5$, or $v_2 = 0.95c$.

1-35 In the Earth frame, Speedo's trip lasts for a time $\Delta t = \frac{\Delta x}{v} = \frac{20.0\text{ ly}}{0.950\ \text{ly/yr}} = 21.05$ Speedo's age advances only by the proper time interval: $\Delta t_p = \frac{\Delta t}{\gamma} = 21.05\text{ yr}\sqrt{1-0.95^2} = 6.574\text{ yr}$ during his trip. Similarly for Goslo, $\Delta t_p = \frac{\Delta x}{v}\sqrt{1-\frac{v^2}{c^2}} = \frac{20.0\text{ ly}}{0.750\ \text{ly/yr}}\sqrt{1-0.75^2} = 17.64\text{ yr}$. While Speedo has landed on Planet X and is waiting for his brother, he ages by

$$\frac{20.0\text{ ly}}{0.750\ \text{ly/yr}} - \frac{0.20\text{ ly}}{0.950\ \text{ly/yr}}\sqrt{1-0.75^2} = 17.64\text{ yr}.$$

Then Goslo ends up older by $17.64\text{ yr} - (6.574\text{ yr} + 5.614\text{ yr}) = 5.45\text{ yr}$.