2-2 (a) Scalar equations can be considered in this case because relativistic and classical velocities are in the same direction.

$$p = \gamma mv = 1.90mv = \frac{mv}{\left[1-(v/c)^2\right]^{1/2}} \Rightarrow \frac{1}{\left[1-(v/c)^2\right]^{1/2}} = 1.90 \Rightarrow v = \left[1-\left(\frac{1}{1.90}\right)^2\right]^{1/2} c$$

$= 0.85c$

(b) No change, because the masses cancel each other.

2-9 (a) When $K = (\gamma - 1)mc^2 = 5mc^2$, $\gamma = 6$ and $E = \gamma mc^2 = 6(0.511\ 0\ \text{MeV}) = 3.07\ \text{MeV}$.

(b) $\frac{1}{\gamma} = \left[1-\left(\frac{v}{c}\right)^2\right]^{1/2}$ and $v = c\left[1-\left(\frac{1}{\gamma}\right)^2\right]^{1/2} = c\left[1-\left(\frac{1}{6}\right)^2\right]^{1/2} = 0.986c$

2-12 (a) When $K_e = K_p$, $m_e c^2(\gamma_e - 1) = m_p c^2(\gamma_p - 1)$. In this case $m_e c^2 = 0.511\ 0\ \text{MeV}$ and $m_p c^2 = 938\ \text{MeV}$, $\gamma_e = \left[1-(0.75)^2\right]^{1/2} = 1.511\ 9$. Substituting these values into the first equation, we find $\gamma_p = 1 + \frac{m_e c^2(\gamma - 1)}{m_p c^2} = 1 + \frac{(0.511\ 0)(1.511\ 9 - 1)}{939} = 1.000\ 279$.

But $\gamma_p = \frac{1}{\left[1-(u_p/c)^2\right]^{1/2}}$; therefore $u_p = c\left(1-\gamma_p^{-2}\right)^{1/2} = 0.023\ 6c$.

(b) When $p_e = p_p$, $\gamma_p m_p u_p = \gamma_e m_e u_e$ or $u_p = \left(\frac{\gamma_e}{\gamma_p}\right)\left(\frac{m_e}{m_p}\right)u_e$,

$$u_p = \left(\frac{1.511\ 9}{1.000\ 279}\right)\left[\frac{0.511\ 0/c^2}{939/c^2}\right](0.75c) = 6.17 \times 10^{-4}c.$$

2-13 (a) $E = 400mc^2 = \gamma mc^2$

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} = 400$$

$$\left(1 - \frac{v^2}{c^2}\right) = \left(\frac{1}{400}\right)^2$$

$$\frac{v}{c} = \left[1 - \frac{1}{400^2}\right]^{1/2}$$

$$v = 0.999\ 997c$$

(b) $K = E - mc^2 = (400 - 1)mc^2 = 399mc^2 = (399)(938.3\ \text{MeV}) = 3.744 \times 10^5\ \text{MeV}$

2-15 (a) $K = \gamma mc^2 - mc^2 = Vq$ and so, $\gamma^2 = \left(1 + \frac{Vq}{mc^2}\right)^2$ and $\frac{v}{c} = \left\{1 - \left(1 + \frac{Vq}{mc^2}\right)^{-2}\right\}^{1/2}$

$$\frac{v}{c} = \left\{1 - \frac{1}{1 + \left(5.0 \times 10^4 \text{ eV}/0.511 \text{ MeV}\right)^2}\right\}^{1/2} = 0.412\,7$$

or $v = 0.413c$.

(b) $K = \frac{1}{2}mv^2 = Vq$

$$v = \left(\frac{2Vq}{m}\right)^{1/2} = \left\{\frac{2\left(5.0 \times 10^4 \text{ eV}\right)}{0.511 \text{ MeV}/c^2}\right\}^{1/2} = 0.442\,c$$

(c) The error in using the classical expression is approximately $\frac{3}{40} \times 100\%$ or about 7.5%

2-19 $\Delta m = 6m_p + 6m_n - m_{\text{C}} = \left[6(1.007\,276) + 6(1.008\,665) - 12\right] \text{ u} = 0.095\,646 \text{ u}$,

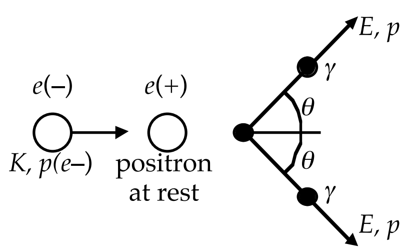
$$\Delta E = (931.49 \text{ MeV/u})(0.095\,646 \text{ u}) = 89.09 \text{ MeV}.$$

Therefore the energy per nucleon $= \frac{89.09 \text{ MeV}}{12} = 7.42 \text{ MeV}$.

2-20 $\Delta m = m - m_p - m_e = 1.008\,665 \text{ u} - 1.007\,276 \text{ u} - 0.000\,548\,5 \text{ u} = 8.404 \times 10^{-4} \text{ u}$

$E = c^2\left(8.404 \times 10^{-4} \text{ u}\right) = \left(8.404 \times 10^{-4} \text{ u}\right)(931.5 \text{ MeV/u}) = 0.783 \text{ MeV}$.

2-21



Conservation of mass-energy requires $K + 2mc^2 = 2E$ where K is the electron's kinetic energy, m is the electron's mass, and E is the gamma ray's energy.

$$E = \frac{K}{2} + mc^2 = (0.500 + 0.511) \text{ MeV} = 1.011 \text{ MeV}.$$

Conservation of momentum requires that $p_{e^-} = 2p\cos\theta$ where p_{e^-} is the initial momentum of the electron and p is the gamma ray's momentum, $\frac{E}{c} = 1.011 \text{ MeV}/c$. Using $E_{e^-}^2 = p_{e^-}^2 c^2 + \left(mc^2\right)^2$ where E_{e^-} is the electron's total energy, $E_{e^-} = K + mc^2$, yields

$$p_{e^-} = \frac{1}{c}\sqrt{K^2 + 2Kmc^2} = \frac{\sqrt{(1.00)^2 + 2(1.00)(0.511)}\ \text{MeV}}{c} = 1.422\ \text{MeV}/c.$$

Finally, $\cos\theta = \frac{p_{e^-}}{2p} = 0.703$; $\theta = 45.3°$.

2-23 In this problem, M is the mass of the initial particle, m_l is the mass of the lighter fragment, v_l is the speed of the lighter fragment, m_h is the mass of the heavier fragment, and v_h is the speed of the heavier fragment. Conservation of mass-energy leads to

$$Mc^2 = \frac{m_l c^2}{\sqrt{1 - v_l^2/c^2}} + \frac{m_h c^2}{\sqrt{1 - v_h^2/c^2}}$$

From the conservation of momentum one obtains

$$(m_l)(0.987\,c)(6.22) = (m_h)(0.868\,c)(2.01)$$
$$m_l = \frac{(m_h)(0.868\,c)(2.01)}{(0.987)(6.22)} = 0.284 m_h$$

Substituting in this value and numerical quantities in the mass-energy conservation equation, one obtains $3.34 \times 10^{-27}\ \text{kg} = 6.22 m_l + 2.01 m_h$ which in turn gives $3.34 \times 10^{-27}\ \text{kg} = (6.22)(0.284) m_l + 2.01 m_h$ or $m_h = \frac{3.34 \times 10^{-27}\ \text{kg}}{3.78} = 8.84 \times 10^{-28}\ \text{kg}$ and $m_l = (0.284) m_h = 2.51 \times 10^{-28}\ \text{kg}$.

2-26 Energy conservation: $\frac{1}{\sqrt{1 - 0^2}} 1\,400\ \text{kg}c^2 + \frac{900\ \text{kg}c^2}{\sqrt{1 - 0.85^2}} = \frac{Mc^2}{\sqrt{1 - v^2/c^2}}$; $3\,108\ \text{kg}\sqrt{1 - \frac{v^2}{c^2}} = M$. Momentum conservation: $0 + \frac{900\ \text{kg}(0.85c)}{\sqrt{1 - 0.85^2}} = \frac{Mv}{\sqrt{1 - v^2/c^2}}$; $1\,452\ \text{kg}\sqrt{1 - \frac{v^2}{c^2}} = \frac{Mv}{c}$.

(a) Dividing gives $\frac{v}{c} = \frac{1\,452}{3\,108} = 0.467$ $\quad v = 0.467\,c$.

(b) Now by substitution $3\,108\ \text{kg}\sqrt{1 - 0.467^2} = M = 2.75 \times 10^3\ \text{kg}$.

2-31 Conservation of momentum γmu:

$$\frac{mu}{\sqrt{1 - u^2/c^2}} + \frac{m(-u)}{3\sqrt{1 - u^2/c^2}} = \frac{Mv_f}{\sqrt{1 - v_f^2/c^2}} = \frac{2mu}{3\sqrt{1 - u^2/c^2}}.$$

Conservation of energy γmc^2:

$$\frac{mc^2}{\sqrt{1-u^2/c^2}}+\frac{mc^2}{3\sqrt{1-u^2/c^2}}=\frac{Mc^2}{\sqrt{1-v_f^2/c^2}}=\frac{4mc^2}{3\sqrt{1-u^2/c^2}}.$$

To start solving we can divide: $v_f = \frac{2u}{4} = \frac{u}{2}$. Then

$$\frac{M}{\sqrt{1-u^2/4c^2}}=\frac{4m}{3\sqrt{1-u^2/c^2}}=\frac{M}{(1/2)\sqrt{4-u^2/c^2}}$$

$$M=\frac{2m\sqrt{4-u^2/c^2}}{3\sqrt{1-u^2/c^2}}$$

Note that when $v \ll c$, this reduces to $M = \frac{4m}{3}$, in agreement with the classical result.

2-33 The energy that arrives in one year is

$$E = \mathscr{P}\,\Delta t = \left(1.79\times10^{17}\ \text{J/s}\right)\left(3.16\times10^{7}\ \text{s}\right) = 5.66\times10^{24}\ \text{J}.$$

Thus, $m = \frac{E}{c^2} = \frac{5.66\times10^{24}\ \text{J}}{\left(3.00\times10^{8}\ \text{m/s}\right)^2} = 6.28\times10^{7}\ \text{kg}$.